

Adaptive Taylor Series and its Applications in the Interval Finite Element Method

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Many engineering problems require solution of the system of equations (usually PDE) with the interval parameters \mathbf{p}

$$A(p)u = f(p) \quad p \in \mathbf{p}. \quad (1)$$

The solution $u = u(t, x, p)$ depend on the time t , special variable x , and the vector of uncertain parameters $p = (p_1, \dots, p_m)$. The interval solution can be defined in the following way

$$\underline{u}(x, t) = \min\{u(x, t, p) : p \in \mathbf{p}\}, \quad \bar{u}(x, t) = \max\{u(x, t, p) : p \in \mathbf{p}\}. \quad (2)$$

Using the Finite Element Method it is possible to solve the equation (1) for each specific value p_0 i.e. $u = u(x, t, p_0)$. In order to get approximate values of the function u around the point $p^{(i)}$ it is possible to apply the Taylor polynomial $P_n(x, t, p, p^{(i)})$. Approximate values of the interval solution can be calculated in the following way

$$\underline{u}_i(x, t) \approx \min\{P_n(x, t, p, p^{(i)}) : p \in \mathbf{p}\} \quad (3)$$

$$\bar{u}_i(x, t) \approx \max\{P_n(x, t, p, p^{(i)}) : p \in \mathbf{p}\} \quad (4)$$

In order to increase the accuracy the solution can be calculated in many points $p^{(i)}$. Numerical examples will be presented during the conference.