Parallel Method for the Solution of Stochastic Differential Equations with the Interval Parameters

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Let us consider the stochastic differential equation with the interval parameters $(p_1, ..., p_m) = p \in \mathbf{p} \subset \mathrm{R}^{\mathbf{m}}$.

$$dX = f(X,p)dt + g(X,p)dW, \quad X(0) = X_0 \tag{1}$$

where X_0 is a random variable, X is a stochastic process, W is a standard Wiener process (standard Brownian motion) f, g are given functions. Important particular case of the equation (1) is the Black-Scholes model $dX = \mu S dt + \sigma S dW$. The solution of the equation (1) is a stochastic process with the interval parameter $X = X(t,p)$. The solution can be also described by the interval stochastic process

$$\underline{X}(t) = min\{X(t,p) : p \in \mathbf{p}\}, \quad \overline{X}(t) = max\{X(t,p) : p \in \mathbf{p}\} \tag{2}$$

In order to get numerical result a discrete version of the equation (1) will be applied (the Euler-Maruyama Method).
Numerical solution can be calculated by using special Monte-Carlo simulations and parallel computing.
The method can be also applied to the solution of stochastic partial differential equations.

Numerical examples (i.e. stochastic ODE and PDE with the interval parameters) will be presented during the conference.