Applications of Parallel Computing to the Solution of Equations with the Random Parameters

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System of equations

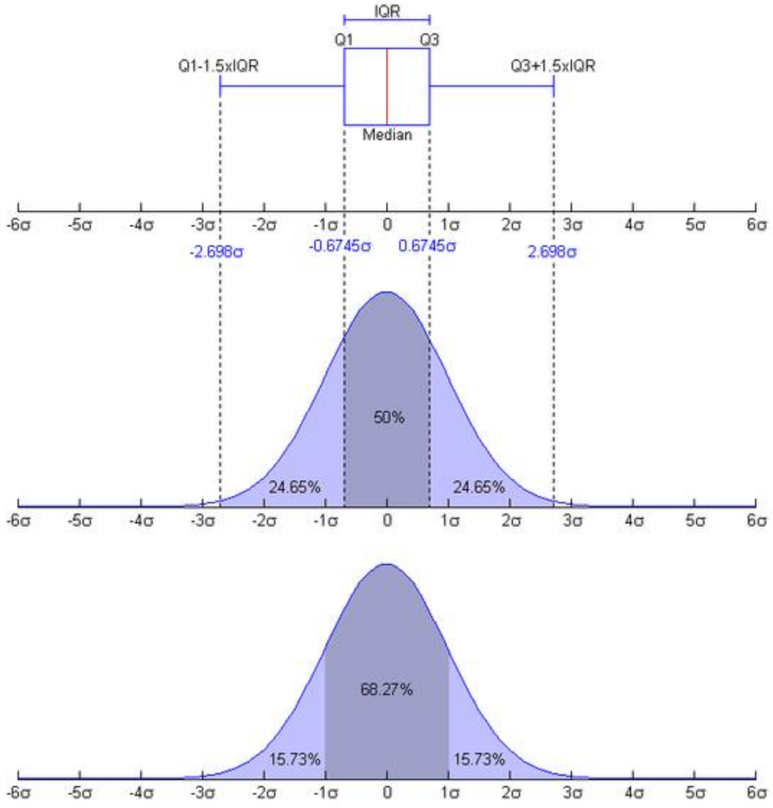
$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

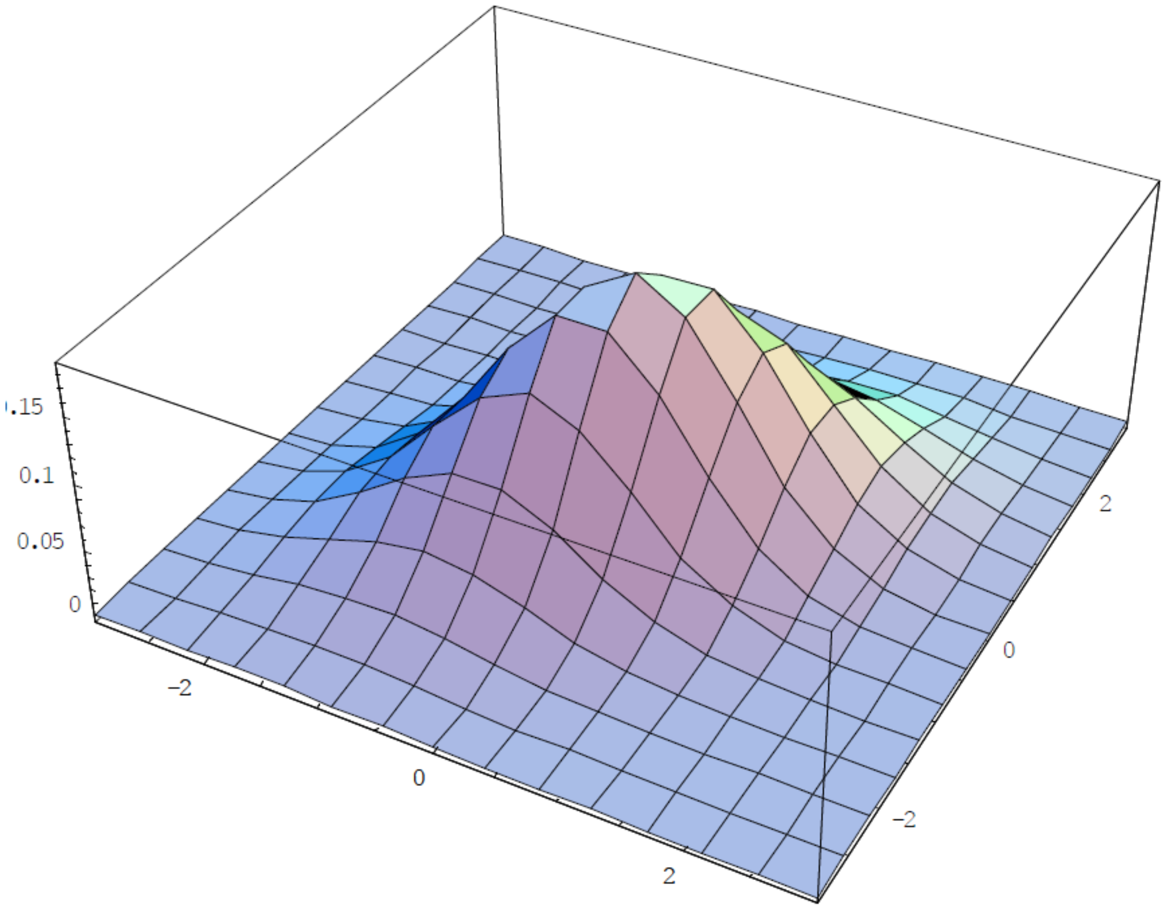
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Random variables

$$X : \Omega \ni \omega \to X(\omega) \in R$$



2D probability density function



System of equations with random parameters

$$\begin{bmatrix} a_{11}(\omega) & a_{12}(\omega) \\ a_{21}(\omega) & a_{22}(\omega) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1(\omega) \\ b_2(\omega) \end{bmatrix}$$

$$x(\omega) = \begin{bmatrix} x_1(\omega) \\ x_2(\omega) \end{bmatrix} = \begin{bmatrix} a_{11}(\omega) & a_{12}(\omega) \\ a_{21}(\omega) & a_{22}(\omega) \end{bmatrix}^{-1} \begin{bmatrix} b_1(\omega) \\ b_2(\omega) \end{bmatrix}$$

Functions of random variables

$$Y(\omega) = g\left(X(\omega)\right)$$

$f_X(x)$ - PDF of variable *X*

$f_Y(y)$ - PDF of variable *Y*

$$f_Y(y) = f_X\left(g^{-1}(y)\right)\left|\frac{dg^{-1}(y)}{dy}\right|$$

Solution of the system of equations is an implicit function

$$x(\omega)=\begin{bmatrix} x_1(\omega) \\ x_2(\omega) \end{bmatrix}=\begin{bmatrix} x_1\left(a_{11}(\omega),a_{12}(\omega),\ldots,b_2(\omega)\right) \\ x_2\left(a_{11}(\omega),a_{12}(\omega),\ldots,b_2(\omega)\right) \end{bmatrix}$$

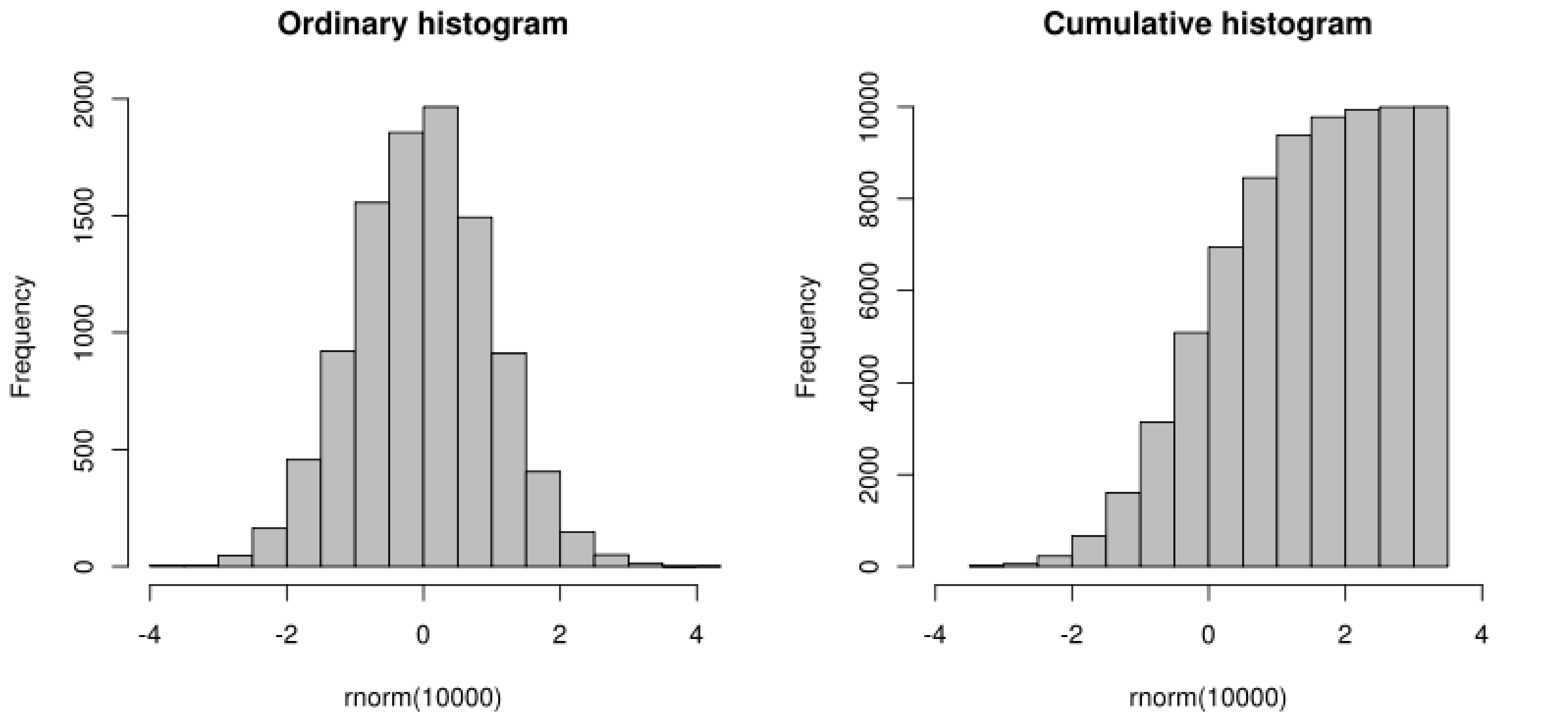
It is very hard to get analytical description of the solution.

Monte Carlo method

$$P(A) = \lim_{N \to \infty} \frac{n_A}{N}$$

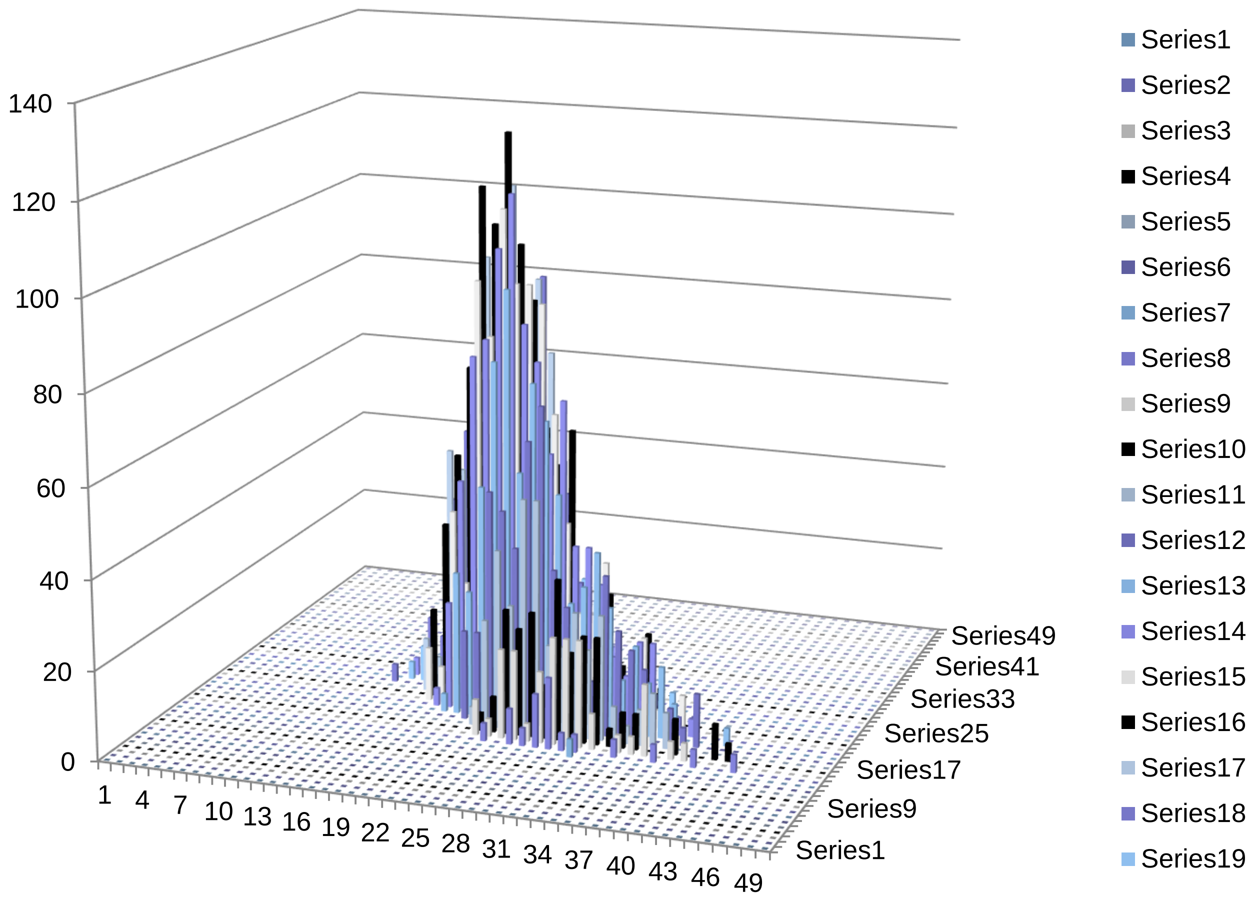
$$P(A) \approx \frac{n_A}{N}$$

Histogram



Ordinary histogram
Frequency
0
500
1000
1500
2000
-4
-2
0
2
4
rnorm(10000)
Cumulative histogram
Frequency
0
2000
4000
6000
8000
10000
-4
-2
0
2
4
rnorm(10000)

2D histogram



Example

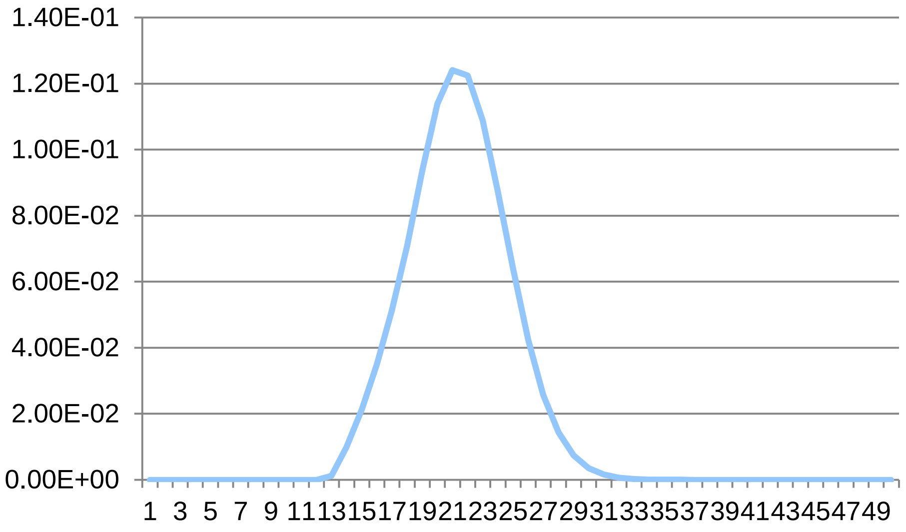
$$\begin{bmatrix} 1+Q & 1+Q \\ 1+Q & -1+Q \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2+Q \\ 0+Q \end{bmatrix}$$

Q - uniformly distributed random variable

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \le x \le b, \\ 0 & \text{for } x < a \text{ or } x > b, \end{cases}$$

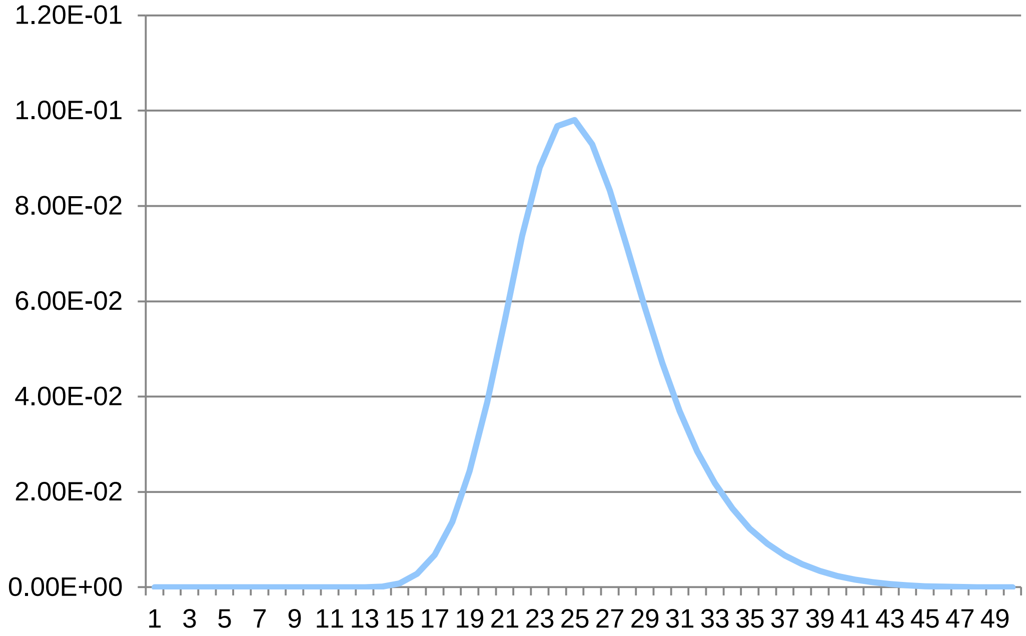
Numerical results

$$X_1 = X_1(\omega)$$



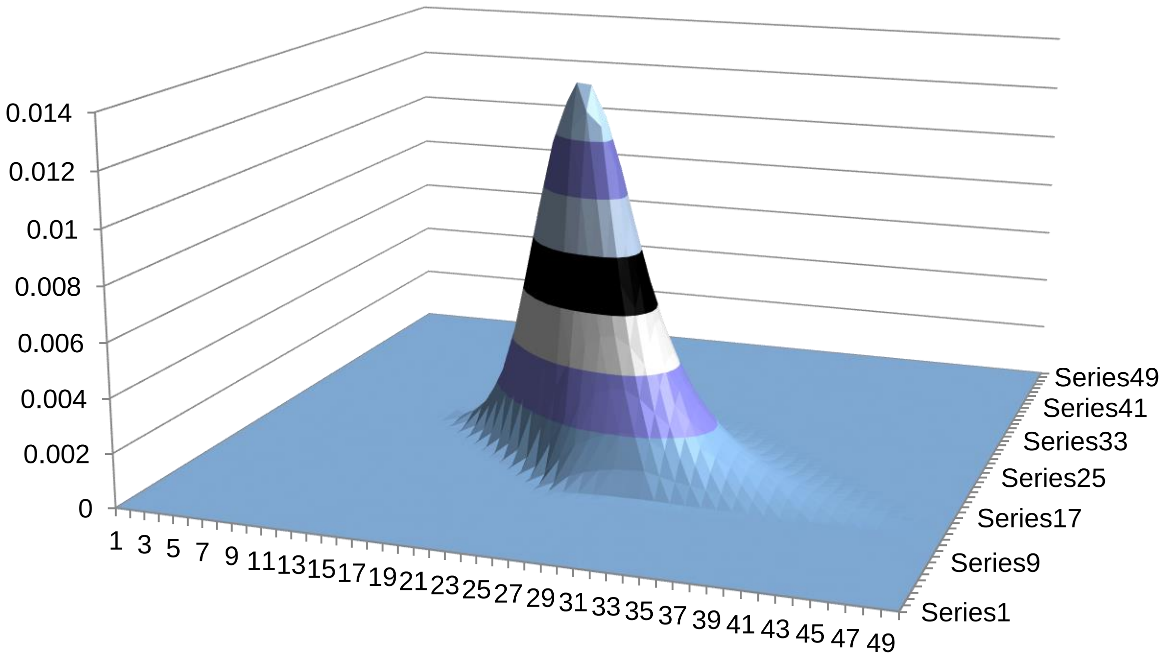
Numerical results

$$X_2 = X_2(\omega)$$



2D PDF

$$\begin{bmatrix} X_1(\omega) \\ X_2(\omega) \end{bmatrix} = X = X(\omega)$$

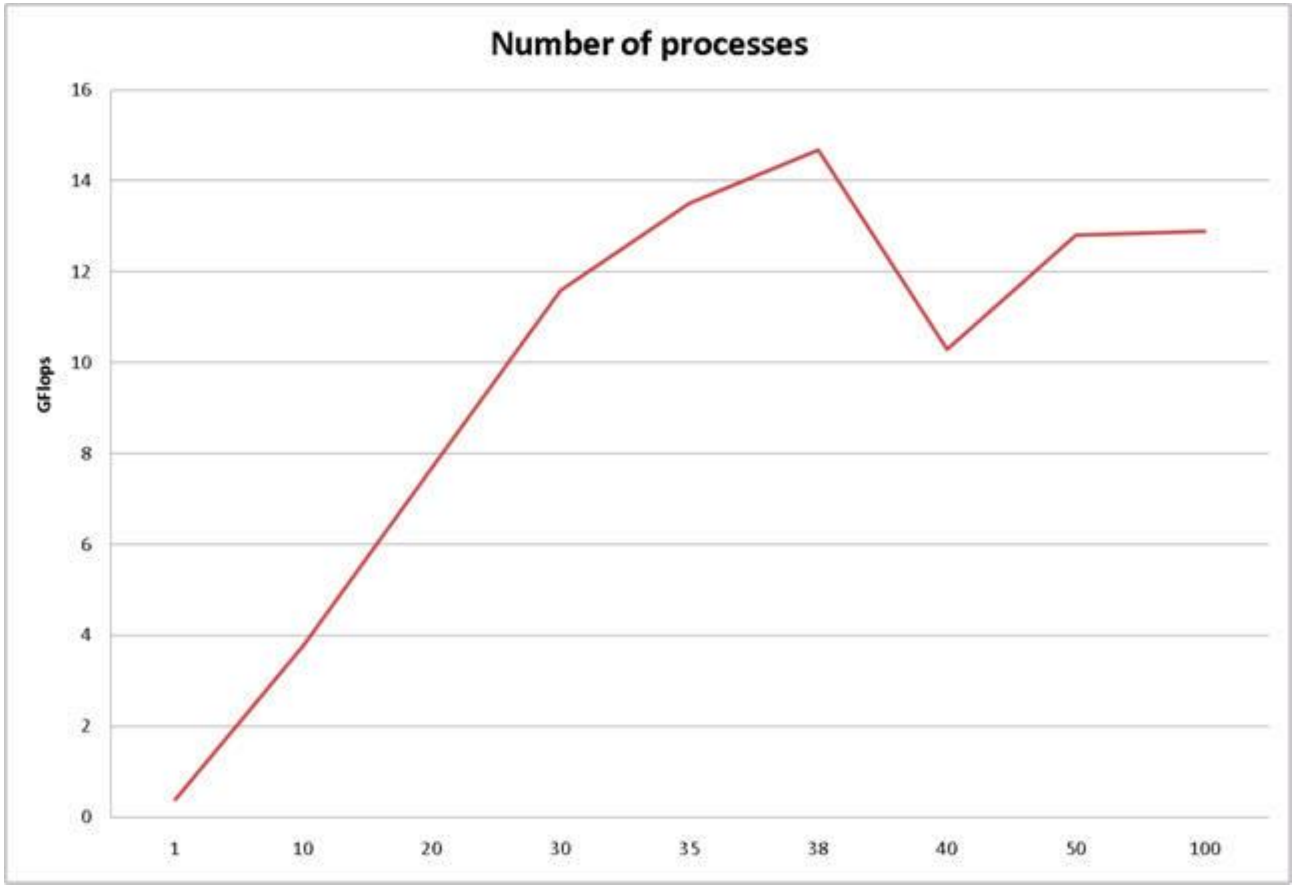


Parallel computing

- MPI_Reduce(histogramX1, histogramX1All, sizeOfTheHistogram, MPI_INT, MPI_SUM, 0, MPI_COMM_WORLD);
- MPI_Reduce(histogramX2, histogramX2All, sizeOfTheHistogram, MPI_INT, MPI_SUM, 0, MPI_COMM_WORLD);

- mpiexec -n 4 progarm 20000000 50

Performance chart for 19 nodes compute cluster



Conclusions

- Using the Monte Carlo Method it is possible to solve system of equations with the random parameters.
- Parallel computing allow us to significantly speed up the calculations and make the calculations more realistic.