Application of order-preserving functions to the modelling of computational mechanics problems with uncertainty

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Abstract

In order to efficiently use mathematical models of engineering structures it is necessary to know the values of the parameters which describe that structures. In this paper the authors will be working on three groups of parameters

1. parameters which can be described by the real numbers (e.g. point loads, displacement of the nodes etc.)
2. parameter which can be described by functions (e.g. distributed loads, density of the materials etc.)
3. parameters which are subsets of R^2 or R^3 (e.g. the shape of the structure).

Unfortunately very often the exact values of these parameters are not known exactly. In such situations it necessary to use some methods for modelling of uncertainty. In the case of parameters which are real numbers it is possible to use intervals [1] (or probability density function). In order to describe the uncertain functional and set parameters it is possible to use interval functions (or stochastic process) and interval sets (or random sets). Functional intervals and interval sets can be defined by

using some partial order relations. According to numerical experiments many solutions of computational mechanics problems are order preserving (i.e. monotone). Because of that extreme values of the solutions can be calculated by using sensitivity analysis [2]. If the sign of the difference $f(y) - f(x)$ and $y - x$ is constant then the extreme values of the function can be calculate by using the extreme values of the variable x. Sometimes it is easier to determine the sign of the infinitesimal increments (or differentials) and calculate the extreme values of the solution in the infinitesimal interval. In that case it is possible to extrapolate the results to the case of narrow intervals.
Presented approach will be applied to the solution of some selected problems of structural and fluid mechanics.

References:

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