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Applications of the Global Optimization Methods for the Solution of Truss Structures with Interval Parameters

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Abstract The problem of solving parametric linear systems whose input data are non-linear functions of interval parameters is considered. The goal is to compute a tight enclosure for the solution set of such systems. Several techniques are employed to reach this goal. Sensitivity analysis is compared with evolutionary optimization method and interval global optimization. Evolutionary optimization is used both to approximate the hull from below and to obtain the starting point for global optimization. Several acceleration techniques are used to speed up the convergence of the global optimization. Additionally, the parallel computations are involved. Some illustrative examples are solved by the discussed methods; the results are compared to literature data produces by other methods. It is shown that interval global optimization can be successfully used for solving the problems under consideration. All optimization methods which are described in this paper are parallelizable and can be implemented using MPI Library.

Keywords: Interval Global Optimization, Sensitivity Analysis, Parametric Linear Systems, Parallel Programming

1. Interval equations

Consider the interval equation

$$F(u, \boldsymbol{p}) = 0 \ , \tag{1}$$

where $\boldsymbol{p} = [\underline{p}_1, \overline{p}_1] \times ... \times [\underline{p}_m, \overline{p}_m]$, $u = (u_1, ..., u_n)$, and $F = (F_1, ..., F_n)$. Function F can be very complicated. One can consider a system of algebraic, differential, integral equations, in general, any type of equations including relational ones.

The solution set of system (1) is defined as

$$u(\boldsymbol{p}) = \{u : F(u, p) = 0, p \in \boldsymbol{p}\} \tag{2}$$

Generally, the solution set $u(\boldsymbol{p})$ has a very complicated shape (it is not necessarily convex). Therefore, the problem of solving system of equations (1) is usually formulated as a problem of finding an interval vector (outer solution) that contains the solution set. The tightest interval solution is called a hull solution [2] or an optimal solution [?]. The problem of computing hull solution can be defined as a family of the following $2n$ global optimization problems:

$$\begin{array}{l} \min\{u_i : F(u, p) = 0, p \in \boldsymbol{p}\} \\ \max\{u_i : F(u, p) = 0, p \in \boldsymbol{p}\} \end{array} , \ \ i = 1, \ldots, n, \tag{3}$$

and the following theorem holds

Theorem 1. Let $F(u,\boldsymbol{p})=0$ and let $\underline{u}_i$ and $\overline{u}_i$ denote, respectively, the solution of the i-th minimization and maximization problem (3). Then

$$\boldsymbol{u}=\Box u(\boldsymbol{p})=\Box\{u:F(u,p)=0,p\in\boldsymbol{p}\}=[\underline{u}_1,\overline{u}_1]\times...\times[\underline{u}_m,\overline{u}_m]. \tag{4}$$

2. Monotonicity and uniform monotonicity

Function $u=u(p_1,p_2,...,p_m)$ is monotonically increasing with the respect to the variable p_i if

$$p_{i0}\geqslant p_{i1}\Rightarrow u(...,p_{i0},...)\geqslant u(...,p_{i1},...) \tag{5}$$

Function $u=u(p_1,p_2,...,p_m)$ is monotonically decreasing with the respect to the variable p_i if

$$p_{i0}\geqslant p_{i1}\Rightarrow u(...,p_{i0},...)\leqslant u(...,p_{i1},...) \tag{6}$$

If the function is monotonically increasing or monotonically decreasing then the function is monotone.

Theorem 2. If the function is monotone with respect to all variables $p_1,...,p_m$ then extreme values of the function $u=u(p_1,...,p_m)$ are attained at vertices of the box $\mathbf{p}$.

In the case of truss structures it is possible to prove that the implicit function $u=u(p_1,...,p_m)$, which is defined by the equation (2) is monotone [3]. Because of that the following theorem is true.

Theorem 3. In the case of truss structures, extreme values of the displacements u are attained at the vertices of the box $\boldsymbol{p}$, where $\boldsymbol{p}$ contain only area of cross-section, Young modulus, and point loads [3].

In order to get extreme values of the displacements $u=u(p_1,...,p_m)$ it is possible to apply endpoint combination method [2]. That is the practical conclusion of the theorem 3. Unfortunately endpoint combination method is very time-consuming, because of that it is not possible to use that method in practice.

Definition 4. If a function $u=u(p)=u(p_1,...,p_m)$ is monotone with respect to all variables $p_1,...,p_m$ for all $p\in\boldsymbol{p}$, then u is uniformly monotone.

Theorem 5. If the function is uniformly monotone, then extreme values can be calculated by using one iteration of the gradient method (sensitivity analysis [9]).

According to the numerical experiments [10] displacements of some truss structures are uniformly monotone and some are not.

For monotone functions $u_i=u_i(p)$ the maximum and the minimum can be found by using the following procedure

$$If\quad \frac{\partial u_i}{\partial p_j}\geqslant 0\ \ then\ \ p_j^{min,i}=\underline{p}_j,\ p_j^{max,i}=\overline{p}_j, \tag{7}$$

$$If\quad \frac{\partial u_i}{\partial p_j}<0\ \ then\ \ p_j^{min,i}=\overline{p}_j,\ p_j^{max,i}=\underline{p}_j, \tag{8}$$

$$\underline{u}_i=u_i(p^{min,i}),\quad \overline{u}_i=u_i(p^{max,i}). \tag{9}$$

Derivatives $\frac{\partial u_i}{\partial p_j}$ and different interval solution can be calculated in parallel.

3. Gradient method for monotone and non-monotone functions

From mathematical point of view the problem of solution of the system of equations with the interval parameters is actually an optimization problem.

$$\underline{u}_i = \begin{cases} min\ u_i \\ K(p)u = Q(p) \\ p \in \mathbf{p} \end{cases}, \quad \overline{u}_i = \begin{cases} max\ u_i \\ K(p)u = Q(p) \\ p \in \mathbf{p} \end{cases}. \tag{10}$$

There are many optimization methods [1], which can be applied to solve the optimization problems (10). One of the simplest methods is the gradient method. It is well known that the maximum growth of the function u is in the direction of the gradient ∇u. In order to find maximum and the minimum of the function it is possible to use this property directly. The method is especially effective for monotone functions. For monotone functions $u_i = u_i(p)$ the maximum and the minimum can be found by using one iteration.

$$If \quad \frac{\partial u_i}{\partial p_j} \geqslant 0 \ \ then \ \ p_j^{min,i} = \underline{p}_j,\ p_j^{max,i} = \overline{p}_j, \tag{11}$$

$$If \quad \frac{\partial u_i}{\partial p_j} < 0 \ \ then \ \ p_j^{min,i} = \overline{p}_j,\ p_j^{max,i} = \underline{p}_j, \tag{12}$$

$$\underline{u}_i = u_i(p^{min,i}), \quad \overline{u}_i = u_i(p^{max,i}). \tag{13}$$

Extreme values of the function can be calculated by using points from the following list

$$L = \{p^{min,1}, p^{min,2}, ..., p^{min,m}, p^{max,1}, p^{max,2}, ..., p^{max,m}\}. \tag{14}$$

Very often some points appear in the list L multiple times. It is possible to create a list of unique points L^*.

$$L^* = \{p^{*1}, p^{*2}, ..., p^{*n}\} \tag{15}$$

In order to get extreme value of the solution it is enough to find the solution in the points from the list L^*

$$\underline{u}_i = min\{u(p^*) : p^* \in L^*\}, \quad \overline{u}_i = max\{u(p^*) : p^* \in L^*\}. \tag{16}$$

Formulas (16) can be applied also in the case when the function $u_i = u_i(p)$ is not monotone. In that case the points $p^{min,i}$ or $p^{max,i}$ are not combinations of endpoints or the interval $\mathbf{p}$ and can be calculated by using general optimization methods. According to many numerical results [9, 5] in many engineering problems the method gives exact results or the accuracy is very good. The method is able to solve large scale engineering problems [9]. Using presented approach it is also possible to solve nonlinear problems of computational mechanics as well as dynamical problems [6]. It is also possible to write general purpose interval FEM software which is based on the gradient method [7].

4. Gradient free method

One of the key aspects of gradient method, described in the previous sections, is calculation of derivatives. Unfortunately calculation of derivatives may be a very complex task in the complex computational methods . However, it is possible to simplify the process of calculations derivatives by using finite difference method. [4].

$$\frac{\partial u_j}{\partial p_i} \approx \frac{u_j(..., p_i + \Delta p_i, ...) - u_j(..., p_i, ...)}{\Delta p_i} \tag{17}$$

It is also possible to apply higher order and multi-point finite difference schemas. In order to calculate the interval solution it is possible to apply the following algorithm.

Algorithm
1) Set $p^0 = (\underline{p}_1, \underline{p}_2, ..., \underline{p}_m)$ and calculate

$$u(p^0) = [K(p^0)]^{-1}Q(p^0) \tag{18}$$

2) For i=1,...,m set $p^i = (\underline{p}_1, ..., \overline{p}_i, ..., \underline{p}_m)$ and calculate

$$u(p^i) = [K(p^i)]^{-1}Q(p^i) \tag{19}$$

3) Calculate approximate values of derivatives of displacements.

$$\frac{\partial u_j}{\partial p_i} \approx \frac{u_j(p^i) - u_j(p^0)}{\Delta p^i} \tag{20}$$

4) Create the initial list of endpoints where the solution is known

$$L^* = \{p^0, p^1, ..., p^m\} \tag{21}$$

5) By using the definition (11,12) define the list different endpoints, which generate extreme values of the displacements L^*_u.
6) Calculate the list of new points $L^* = L^* \cup L^*_u$. For all new points in the list L^* solve the system of equation $Ku = Q$ and find appropriate values necessary in postprocessing.

7) Interval displacements can be calculated in the following way

$$\underline{u}_i = min\{u_i(p) : p \in L^*\}, \quad \overline{u}_i = max\{u_i(p) : p \in L^*\}. \tag{22}$$

The algorithm described above can be applied to any sufficiently regular linear and non-linear problem of computational mechanics. The method makes use of only the values of the solution at some points. Thus it is possible to use the results generated by the use of existing engineering software.

5. Combinatoric solution

Lets consider 11 bar truss which is shown on the Fig. 1, 2, 3. Numerical data are the following: Young modulus $E \in [1.9 \cdot 10^{11}, 2.1 \cdot 10^{11}]$ [Pa], area of cross section $A = 0.0001[m^2]$ [m], point load $P \in [-1050, -950]$ [N], width L=1 [m], height H= 1 [m]. Nodes and appropriate degree of freedom (DOF) are shown in the Table 1. In calculations the combinatoric method was applied. Results which are given by the combinatoric method are exact and can be used for comparison with the other methods of calculation.

Relations between the interval displacements and the uncertainty are shown on the Fig. 4-7. As we can see this relation is nonlinear and the uncertainty of the results is a nonlinear function of the uncertainty of the data.

Using the exact solution it is possible to check monotonicity of the solution as a function of selected variables. On the Fig. ?? it is possible to see the relation between the Young modulus E_3 and E_4 and the displacement u_2. As we can see the relation is monotone. Similar graph can be created for all solution and all parameters. All these relations are monotone. Unfortunately that doesn't imply the uniform monotonicity.

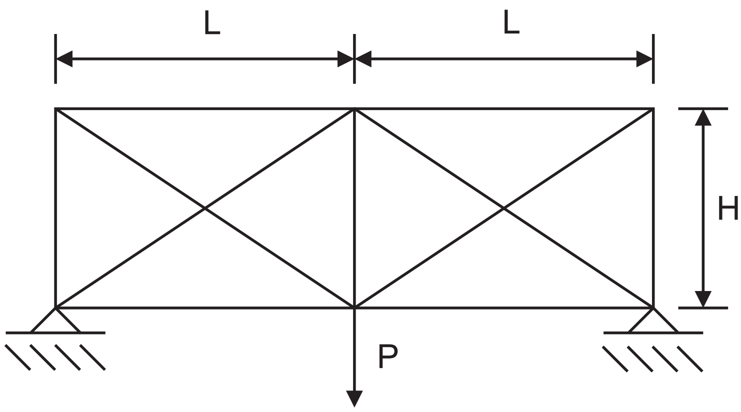

Figure 1. 11 bar truss

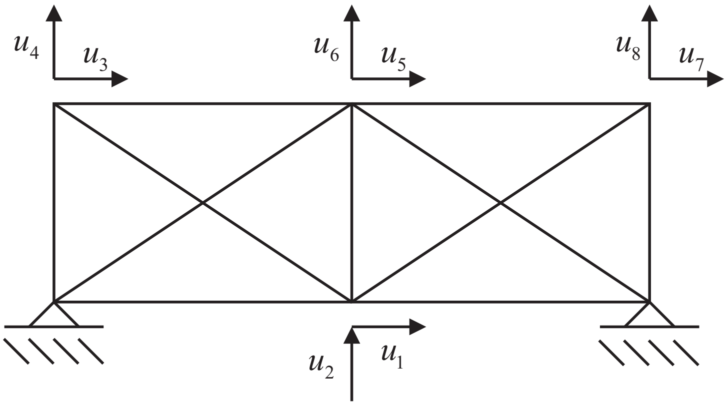

Figure 2. 11 bar truss - displacements

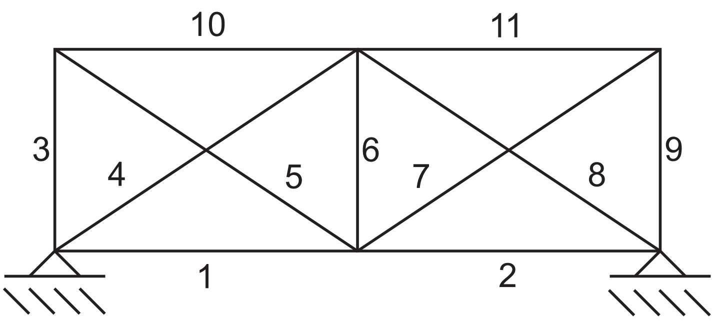

Figure 3. 11 bar truss - elements

6. Gradient free method (H≠L)

Let us consider the same truss structure but with H=5 $[m]$, and L= 10 $[m]$. In this section the gradient free method will be applied. According to the numerical results (compare table 2, 3) 2 displacements (u_1 and u_5) were calculated with the 10% of error. Additionally according to the Table 5 many endpoints of the parameters were predicted incorrectly. In almost all displacements the dependence on parameters E_1, E_2, E_6 is not monotone.
In order to detect non-monotonicity second order test can be applied. It is possible to calculate derivatives by using the same method like displacements (i.e. gradient free method). For the displacement u_5 the results are shown in the Table 6. According to the Table 6 displacement u_5 is not monotone with respect to the variables E_1, E_2, E_6 and P because the sign of the

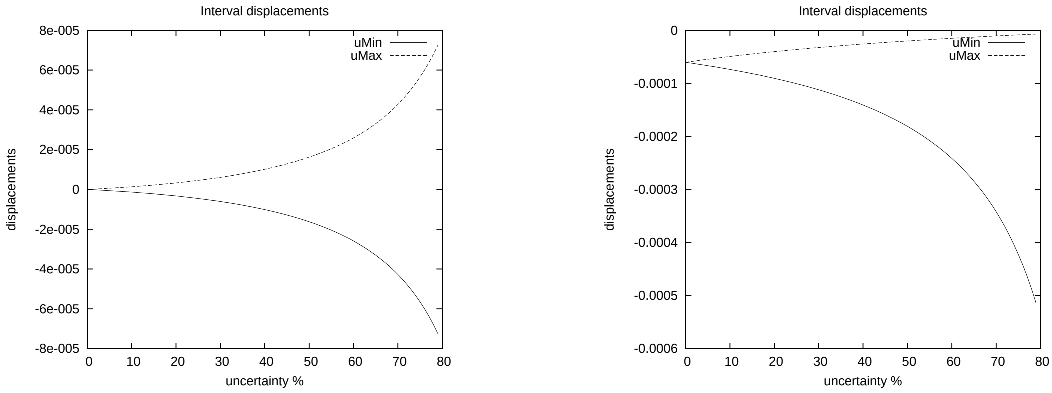

Figure 4. Uncertainty in displacements: u_1 and u_2.

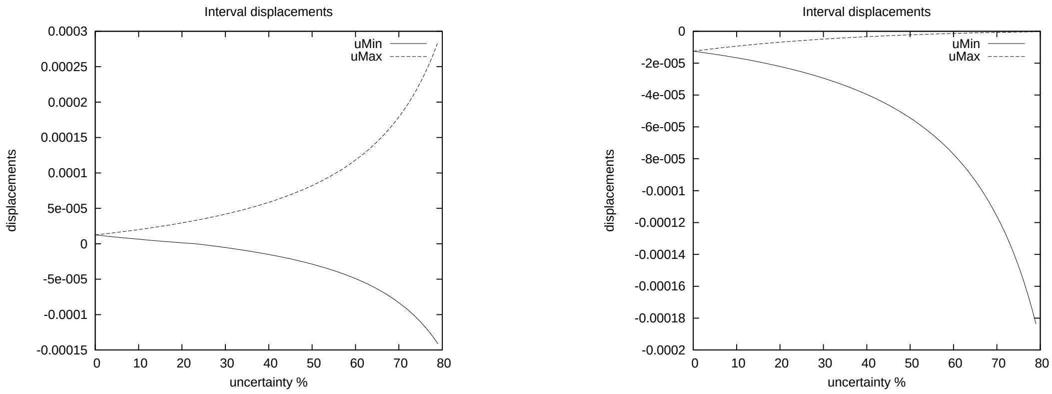

Figure 5. Uncertainty in displacements: u_3 and u_4.

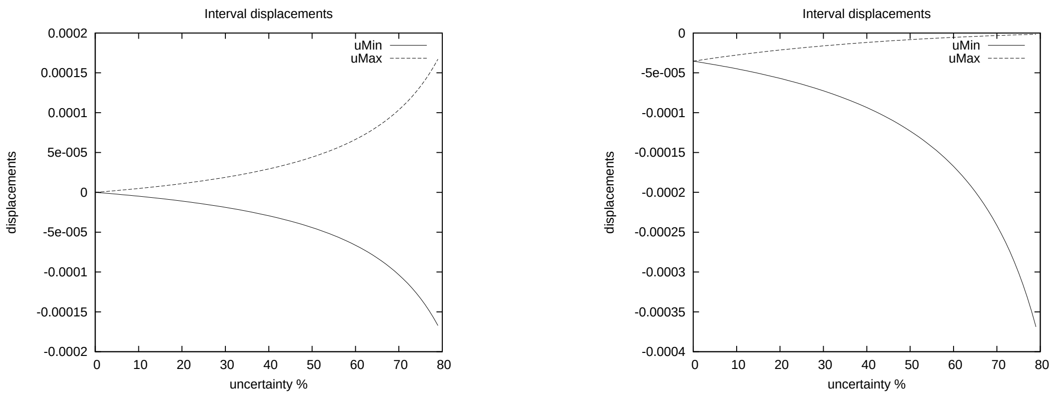

Figure 6. Uncertainty in displacements: u_5 and u_6.

derivatives is not constant. As we can see by using second order monotonicity test it is possible find non-monotone relations between displacements and the uncertain parameters. In order to find more accurate solution in the case of non-monotone parameters more accurate optimization methods can be applied [1]. These methods (as well as second order monotonicity tests) are not implemented yet and will be a topic of our future research.

Table 1. Nodes in 11 bar truss.

Nodes	x [m]	y [m]	u_x (DOF)	u_y (DOF)
1	0	0	-	-
2	10	0	1	2
3	20	0	-	-
4	0	5	3	4
5	10	5	5	6
6	20	5	7	8

Table 2. Displacement in 11 bar truss 5% uncertainty (lower bound).

	Combinatoric	Gradient free	Error
DOF	$\underline{u}$ [m]	$\underline{u}$ [m]	$\underline{u}$ %
1	-1.538050E-04	-1.388490E-04	9.724001E+00
2	-1.652290E-02	-1.652290E-02	0.000000E+00
3	2.440140E-03	2.440140E-03	0.000000E+00
4	-8.664030E-04	-8.664030E-04	0.000000E+00
5	-3.510260E-04	-3.172980E-04	9.608405E+00
6	-1.409620E-02	-1.409620E-02	0.000000E+00
7	-3.537200E-03	-3.537200E-03	0.000000E+00
8	-8.664030E-04	-8.664030E-04	0.000000E+00

Table 3. Displacement in 11 bar truss 5% uncertainty (upper bound).

	Combinatoric	Gradient free	Error
DOF	$\overline{u}$ [m]	$\overline{u}$ [m]	$\overline{u}$ %
1	1.538050E-04	1.420150E-04	7.665551E+00
2	-1.352560E-02	-1.352560E-02	0.000000E+00
3	3.537200E-03	3.536200E-03	2.827095E-02
4	-6.318360E-04	-6.318520E-04	2.532303E-03
5	3.510260E-04	3.506350E-04	1.113878E-01
6	-1.140120E-02	-1.140120E-02	0.000000E+00
7	-2.440140E-03	-2.440960E-03	3.360463E-02
8	-6.318360E-04	-6.318520E-04	2.532303E-03

Table 4. Binary code of the interval displacements in 11 bar truss for 5% uncertainty (lower bound).

	Combinatoric	Gradient free
DOF	$\underline{u}$	$\underline{u}$
1	0,0,1,0,1,0,0,1,0,0,1,0	0,0,1,0,1,1,0,1,0,0,1,1
2	0,0,0,0,0,0,0,0,0,0,0,0	0,0,0,0,0,0,0,0,0,0,0,0
3	0,0,1,1,0,1,1,0,0,1,1,1	0,0,1,1,0,1,1,0,0,1,1,1
4	0,0,1,1,0,0,1,1,0,1,0,0	0,0,1,1,0,0,1,1,0,1,0,0
5	1,1,0,1,0,1,1,0,0,1,1,0	0,0,0,1,0,1,1,0,0,1,1,1
6	1,0,0,0,0,1,0,0,0,0,0,0	0,0,0,0,0,1,0,0,0,0,0,0
7	0,0,0,0,0,0,1,0,0,1,1,0	0,0,0,0,0,0,1,0,0,1,1,0
8	0,0,1,1,1,0,0,1,0,1,0,0	0,0,1,1,1,0,0,1,0,1,0,0

Table 5. Binary code of the interval displacements in 11 bar truss for 5% uncertainty (upper bound).

	Combinatoric	Gradient free
DOF	$\overline{u}$	$\overline{u}$
1	0,0,0,1,0,0,1,0,1,1,0,0	1,1,0,1,0,0,1,0,1,1,0,0
2	1,1,1,1,1,1,1,1,1,1,1,1	1,1,1,1,1,1,1,1,1,1,1,1
3	0,0,0,0,1,0,0,1,1,0,0,0	1,1,0,0,1,0,0,1,1,0,0,0
4	0,0,0,0,1,1,0,0,1,0,1,1	1,1,0,0,1,1,0,0,1,0,1,1
5	1,1,1,0,1,1,0,1,1,0,0,0	1,1,1,0,1,0,0,1,1,0,0,0
6	1,1,1,1,1,0,1,1,1,1,1,1	1,1,1,1,1,0,1,1,1,1,1,1
7	0,0,1,1,1,1,0,1,1,0,0,1	1,1,1,1,1,1,0,1,1,0,0,1
8	0,0,0,0,0,1,1,0,1,0,1,1	1,1,0,0,0,1,1,0,1,0,1,1

Table 6. Derivative of the displacement u_5 with respect to Young's modulus and point load.

Derivative	Min	Max	Sign
$\frac{\partial u_2}{\partial E_1}$	-3.26934E-16	2.508830E-16	?
$\frac{\partial u_2}{\partial E_2}$	-3.26934E-16	2.50883E-16	?
$\frac{\partial u_2}{\partial E_3}$	-1.92976E-15	-1.24505E-15	$-$
$\frac{\partial u_2}{\partial E_4}$	1.24505E-15	1.92976E-15	$+$
$\frac{\partial u_2}{\partial E_5}$	-4.97695E-16	-3.01577E-16	$-$
$\frac{\partial u_2}{\partial E_6}$	-1.15435E-16	8.06621E-17	?
$\frac{\partial u_2}{\partial E_7}$	3.01578E-16	4.97696E-16	$+$
$\frac{\partial u_2}{\partial E_8}$	-2.6533E-15	-1.76977E-15	$-$
$\frac{\partial u_2}{\partial E_9}$	2.53431E-15	3.60947E-15	$+$
$\frac{\partial u_2}{\partial E_{10}}$	1.76977E-15	2.6533E-15	$+$
$\frac{\partial u_2}{\partial E_{11}}$	-3.61005E-15	-2.53463E-15	$-$
$\frac{\partial u_2}{\partial E_P}$	-9.13142E-09	9.93587E-09	?

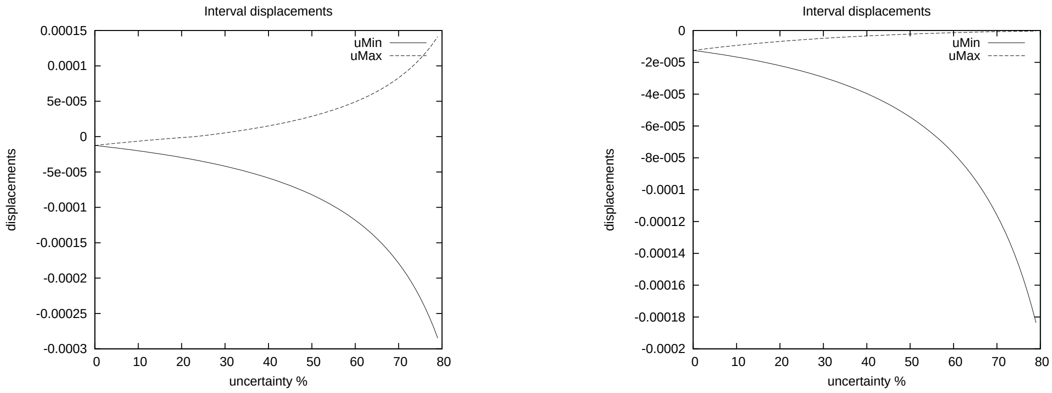

Figure 7. Uncertainty in displacements: u_7 and u_8

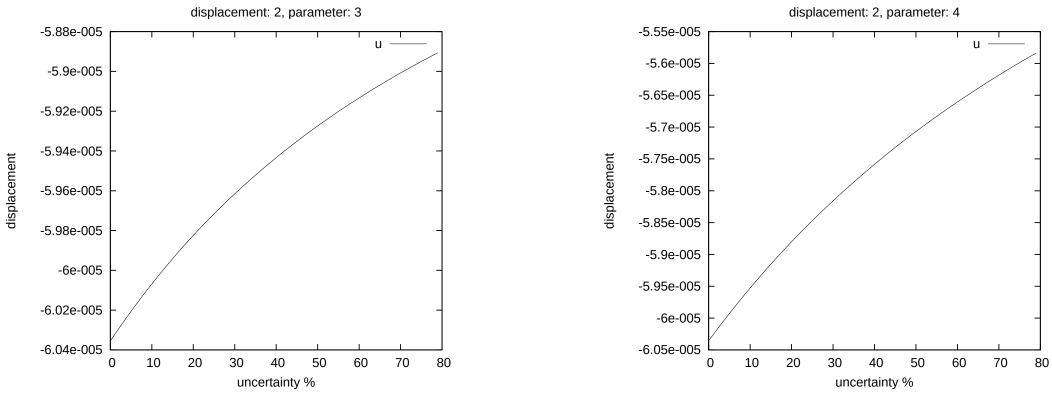

Figure 8. Relation between the displacements u_2 and Young's modulus E_3 and E_4

7. Gradient method

In order to increase accuracy it is possible to apply the gradient method to each endpoint separately. In this example we assume that $L = 1$ $[m]$, $H = 1$ $[m]$. For 5% uncertainty the exact solution was calculated in the second interaction. For 80% uncertainty the exact solution was calculated in the third interaction. This numerical experiment indicate that the problem is not uniformly monotone however it is possible to take advantage from monotonicity with respect to each parameter separately and get the exact solution after several steps of gradient method. Accuracy of the method is shown in the Table

Table 7. Interval data (uncertainty 5%)

Name	Value	Units
E	[190000000000, 210000000000]	$[\frac{N}{m^2}]$
A	0.0001	$[m^2]$
P	[950, 1050]	$[N]$
L	1	$[m]$
H	1	$[m]$

Table 8. First iteration (5% uncertainty).

u	Min	Max
0	-5.48192006298979E-07	5.57569204411639E-07
1	-6.67085326445197E-05	-5.46072115298676E-05
2	9.3090959916984E-06	1.60451258356066E-05
3	-1.44144556825617E-05	-1.08127734148746E-05
4	-2.2952657944851E-06	2.06813835385586E-06
5	-3.9863234281895E-05	-3.13056777887473E-05
6	-1.60434451955151E-05	-9.30757766827653E-06
7	-1.44176417565612E-05	-1.08151874177714E-05

Table 9. Second iteration 5% uncertainty

u	Min	Max
0	-6.09181986301072E-07	6.09181986301072E-07
1	-6.67085326445197E-05	-5.46072115298676E-05
2	9.30618787114789E-06	1.60451258356066E-05
3	-1.44205576948356E-05	-1.08127734148746E-05
4	-2.2952657944851E-06	2.2952657944851E-06
5	-3.9863234281895E-05	-3.13056777887473E-05
6	-1.60451258356066E-05	-9.30618787114789E-06
7	-1.44205576948356E-05	-1.08127734148746E-05

Table 10. Interval data (uncertainty 80%)

Name	Value	Units
E	[40000000000, 360000000000]	$\left[\frac{N}{m^2}\right]$
A	0.0001	$[m^2]$
P	[200, 1800]	$[N]$
L	1	$[m]$
H	1	$[m]$

Table 11. First iteration (80% uncertainty).

u	Min	Max
0	-7.94352515528512E-06	1.04085337294952E-05
1	-0.000543198051533946	-6.70614878436971E-06
2	-1.65901870368457E-05	0.000302585422522513
3	-0.000183199835552243	-3.07029992187359E-07
4	-0.000177807675189214	1.79549348567756E-05
5	-0.000389993956419502	-1.47857979313035E-06
6	-0.000301333482883328	1.6755662292323E-05
7	-0.000193590188755261	-3.27875350856272E-07

Table 12. Second iteration (80% uncertainty).

u	Min	Max
0	-7.74371476016005E-05	7.27236848662643E-05
1	-0.000543198051533946	-6.70614878436971E-06
2	-0.000151050521024827	0.000302585422522513
3	-0.000194959365931795	-3.07029992187359E-07
4	-0.000177807675189214	0.000177807675189214
5	-0.000389993956419502	-1.47857979313035E-06
6	-0.000302585422522513	0.000151050521024827
7	-0.000194959365931795	-3.07029992187359E-07

Table 13. Third iteration (80% uncertainty).

u	Min	Max
0	-7.74371476016005E-05	7.74371476016005E-05
1	-0.000543198051533946	-6.70614878436971E-06
2	-0.000151050521024827	0.000302585422522513
3	-0.000194959365931795	-3.07029992187359E-07
4	-0.000177807675189214	0.000177807675189214
5	-0.000389993956419502	-1.47857979313035E-06
6	-0.000302585422522513	0.000151050521024827
7	-0.000194959365931795	-3.07029992187359E-07

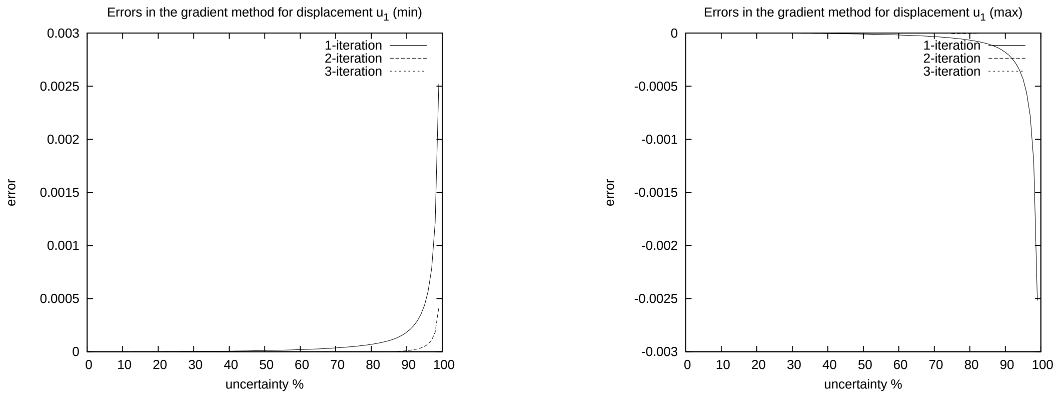

Figure 9. Error in the gradient method for different number of iterations for the displacement u_1

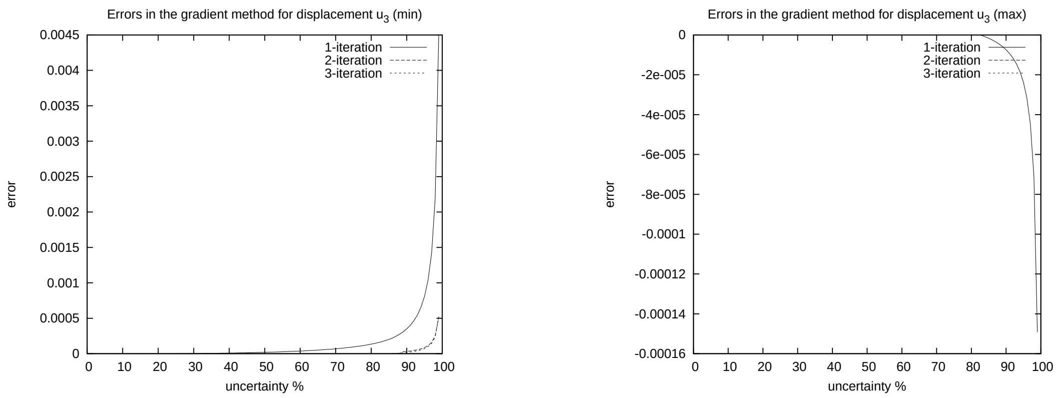

Figure 10. Error in the gradient method for different number of iterations for the displacement u_3

8. Evolutionary algorithm description

Global optimization problems (3) can be solved using evolutionary methods. The result obtained with evolutionary optimization approximates from below the hull solution.

Each evolutionary algorithm require some input parameters. These are: population size, crossover rate, mutation rate, and number of generations. All of them have great influence on the result of the optimization, but the choice of the best values is still a matter of trial. Suggestions for parameter values can be found in the literature.

In this approach, elements of the initial population are generated randomly based on the uniform distribution. The 10% of the best individuals pass to the next generation, and the rest of population is generated using a non-uniform mutation and arithmetic crossover. It turns out from numerical experiments that mutation should be applied to the variables of individuals with probability close to one, while the crossover rate should be less than 0.3. Population size, number of generations depends strongly on the size problem.

Parallel implementations of evolutionary algorithms come in two group. Coarse grained parallel genetic algorithms assume a population on each of the computer nodes and migration of individuals among the nodes. Fine grained parallel genetic algorithms assume an individual on each processor node which acts with neighbouring individuals for selection and reproduction. Other variants, like genetic algorithms for online optimization problems, introduce time-dependence or noise in the fitness function.

9. Global optimization method

The strategy described in Section 8 produces very good inner approximation of the actual hull solution. In order to get reliable solution an interval global optimization method can be applied [11]. The main deficiency of the global optimization is high computational complexity. To cope with this problem, several acceleration techniques are used. The monotonicity test is performed, various inclusion functions are used, cut-off test based on the result of evolutionary optimization is performed, and the technique which deals with parallel computations is employed. Many acceleration techniques can be applied in parallel.

10. Summary

In this paper different optimization methods are applied for solution of system of equations with the interval parameters. The objective is to find the best optimization algorithm that can be applied for each specific problem.

Gradient methods are very fast and they give the exact results if the problem is uniformly monotone. Unfortunately, very often the problems are not uniformly monotone and in that case gradient methods give a very good inner estimation of the optimal solution.

Evolutionary algorithms are stochastic optimization methods which can be applied in situations where the gradient method (based on monotonicity assumption) gives inaccurate results. The evolutionary optimization result also approximates the optimal solution from below. However, usually this approximation is more accurate for non-monotone problems. Moreover, the evolutionary optimization result can be used to perform an efficient cut-off test for global optimization.

In order to get reliable solution special global optimization method is proposed [11]. Suggested acceleration techniques significantly reduce the computational time of global optimization.

According to numerical results, the gradient (or gradient free) method give very accurate solution [10] for many problems of structural mechanics. However, there is also a large class of problems for which monotonicity assumption is not acceptable. In that cases different optimization methods such as evolutionary optimization or interval global optimization can be applied.

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