Constraints-Related Set Computations: A New FEM-Motivated Approach to Propagating Uncertainty

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Abstract

In applications of FEM to science and engineering, it is often crucial to gauge the accuracy of the computational results. There exist many tools and techniques for gauging *discretization* accuracy. In many cases, however, there is another important source of inaccuracy: the fact that we usually only know the initial values and the parameters of the system with some uncertainty. Often, we only know the bounds on the inaccuracy: e.g., in engineering applications, we usually have tolerances on the sizes and on the characteristics of the corresponding materials. In such situations, we only know intervals of possible values of the corresponding parameters, and we need to "propagate" these interval through the FEM algorithms to produce intervals of possible values of the results.

In the traditional approach to such "interval computations", at each intermediate stage of the computation, we have intervals of possible values of the corresponding quantities [3]. The problem is that several intermediate values r_i, r_j come from the same quantities and are thus related. If we do not take this relation into account, we get intervals with excess width. To take this relation into consideration, in addition to intervals $\mathbf{r}_i$ of possible values of the quantities r_i, we also keep sets $\mathbf{r}_{ij} \subset \mathbf{r}_i \times \mathbf{r}_j$ of possible values of pairs (r_i, r_j) (sets of triples, etc.).

To perform such "set computations", we can enclose sets into collections of finite elements [1]. Even for grids, this idea has led to feasible (polynomial time) algorithms for estimating statistics (variance, correlation, etc.) and for solving ordinary differential equations (ODEs) with given accuracy [2]. In this talk, we show that the use of adaptive finite elements can lead to even more efficient algorithms, and we describe potential engineering applications.

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References

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