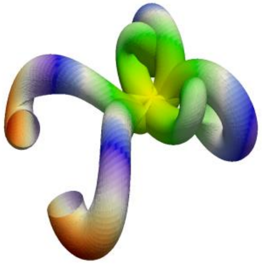
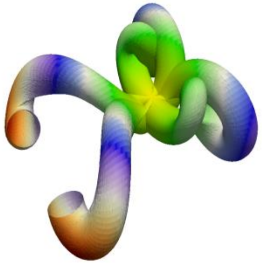
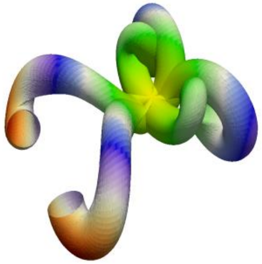
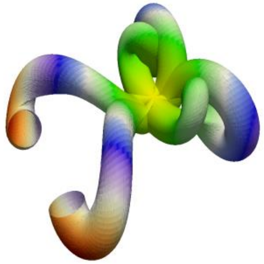
Andrzej Pownuk

Calculating Risk of Cost in Civil Engineering Projects by Using Imprecise Probability and HPC Computing

http://andrzej.pownuk.com

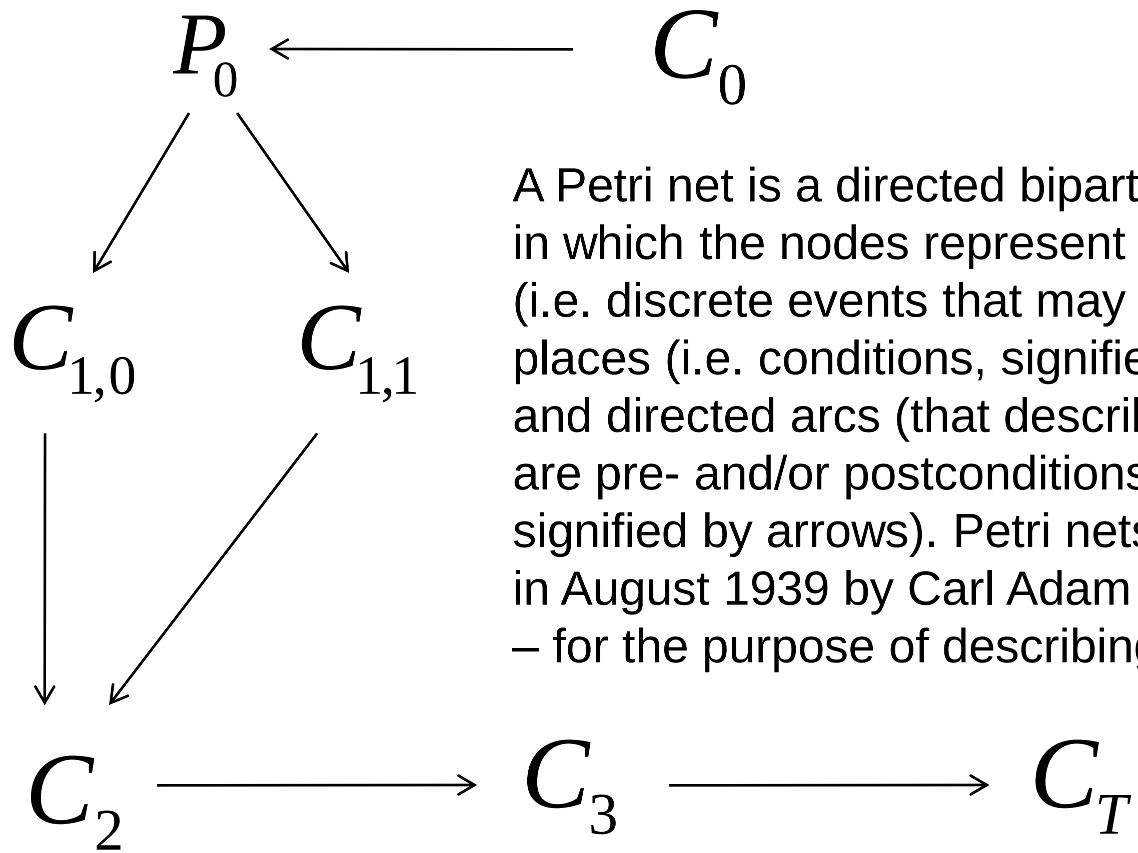





Total cost

- Partial costs C_i
  - cost of concrete
  - cost of labor
  - etc.

- Total cost $C_T = \sum_i C_i$

Petri networks



A Petri net is a directed bipartite graph, in which the nodes represent transitions (i.e. discrete events that may occur, signified by bars), places (i.e. conditions, signified by circles), and directed arcs (that describe which places are pre- and/or postconditions for which transitions, signified by arrows). Petri nets were invented in August 1939 by Carl Adam Petri – at the age of 13 – for the purpose of describing chemical processes.

Uncertainty

Risk us an integral part of each civil engineering project.
We can define it as possibility of occurrence of loss.
One of the most popular type contracts in Poland is
(guaranteed maximum price or cost contract) .

At this time task and costs are predicted on the basis on deterministic unit costs. Tasks and unit costs are deterministic.

Unfortunately, in reality tasks and unit costs may change
because of the influence of different and usually uncertain factors.

Uncertainty

Sources of uncertainty:

- civil engineering projects are very unique,
- imprecise measurements,
- imprecise weather,
- imprecise geology,
- imprecise schedules etc.

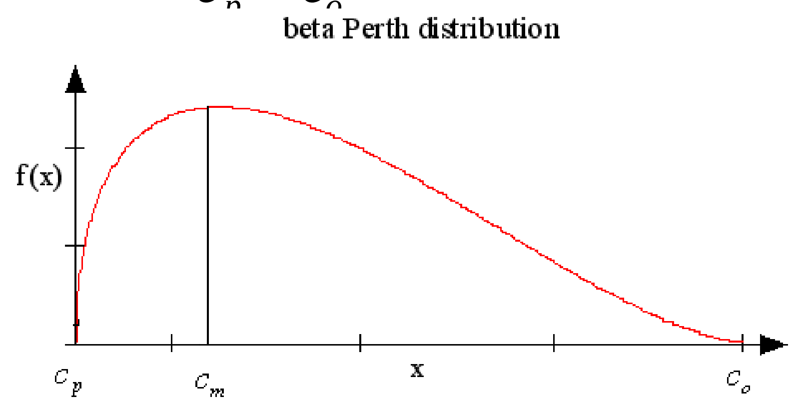
Beta distribution

$$f_{\alpha_F,\beta_F}(x) = \frac{\Gamma(\alpha_F+\beta_F)}{\Gamma(\alpha_F)\Gamma(\beta_F)}(1-x)^{\beta_F-1}x^{\alpha_F-1}, \qquad x\in[0,1]$$

$$E(c) = \frac{c_o + 4\cdot c_m + c_p}{6}$$

$$\beta = 4-\alpha \qquad\qquad \alpha = \frac{4\cdot(c_m-c_o)}{c_p-c_o}$$

$$f_{\alpha,\beta}(x) = \frac{1}{c_o-c_p}\, f_{\alpha,\beta}\left(\frac{x-c_p}{c_o-c_p}\right)$$

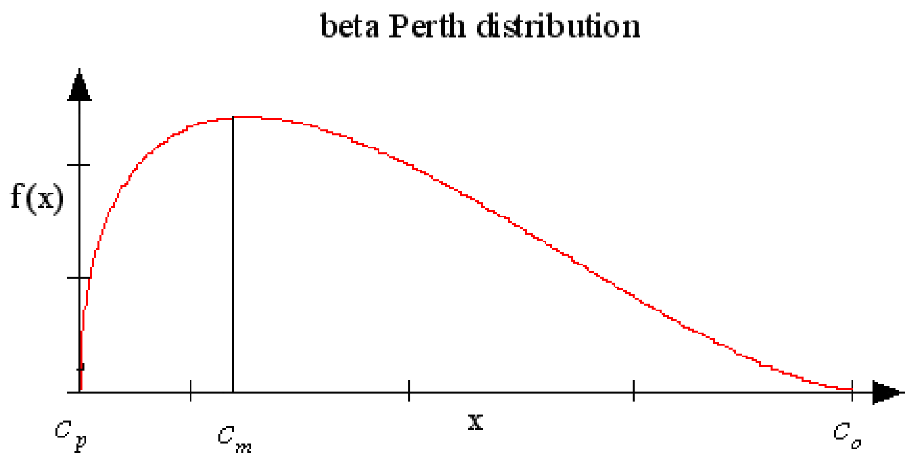


PDF defined by given costs

$$E(c) = \frac{c_o + 4 \cdot c_m + c_p}{6}$$

$$\beta = 4 - \alpha \qquad \alpha = \frac{4 \cdot (c_m - c_o)}{c_p - c_o}$$

$$f_{\alpha,\beta}(x) = \frac{1}{c_o - c_p} f_{\alpha,\beta}\left(\frac{x - c_p}{c_o - c_p}\right)$$



What is the risk?

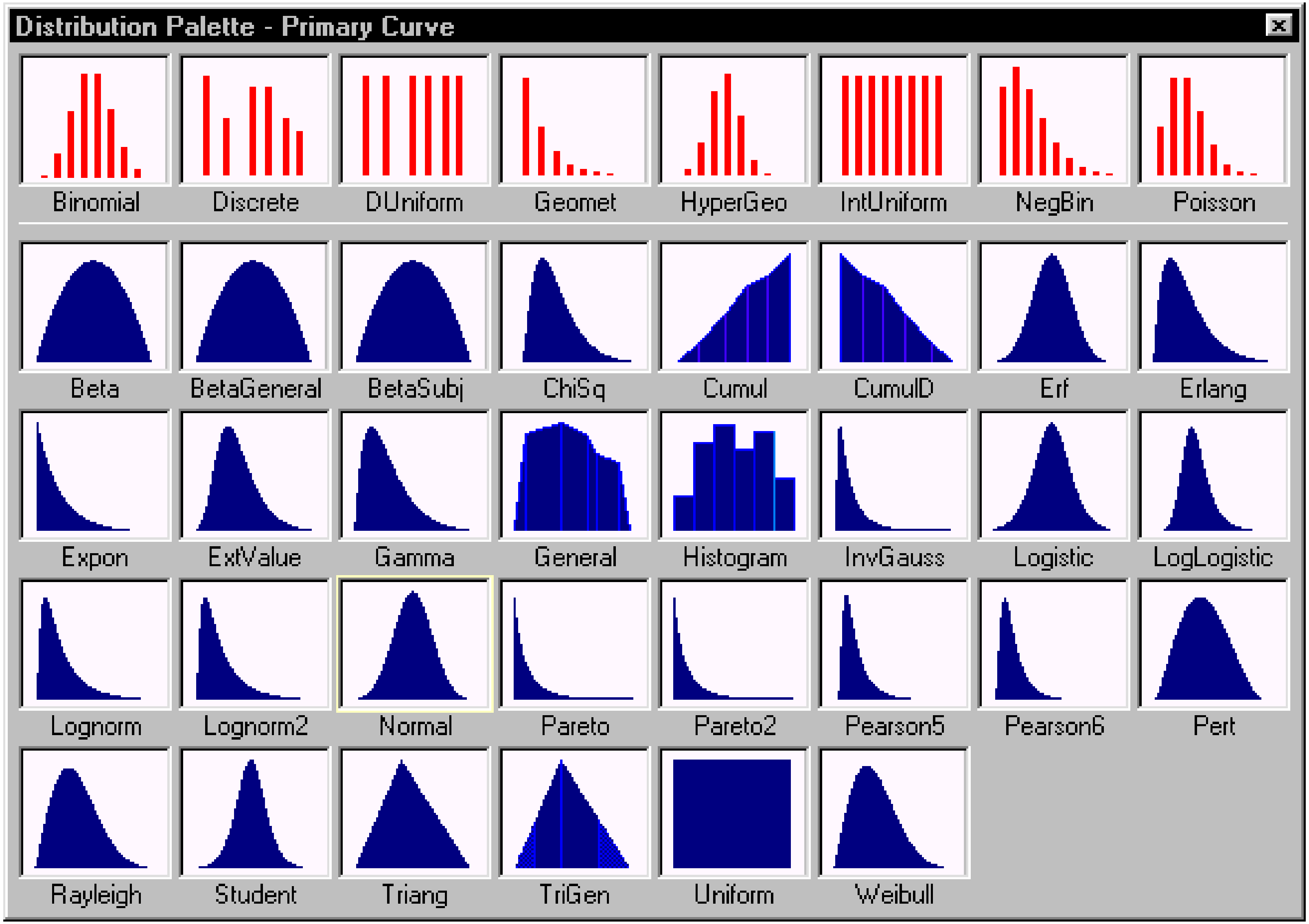
- Estimated cost C_0
- Real cost C

$$C \leq C_0$$

$$C > C_0$$

$$P_f = R = P\{C > C_0\}$$

Pure probabilistic approach



Bayesian inference (subjective probability)

$$P(H \mid E) = \frac{P(E \mid H)\,P(H)}{P(E)}$$

$P(H)$ - is called the *prior probability* of *H* that was inferred before new evidence, *E*, became available

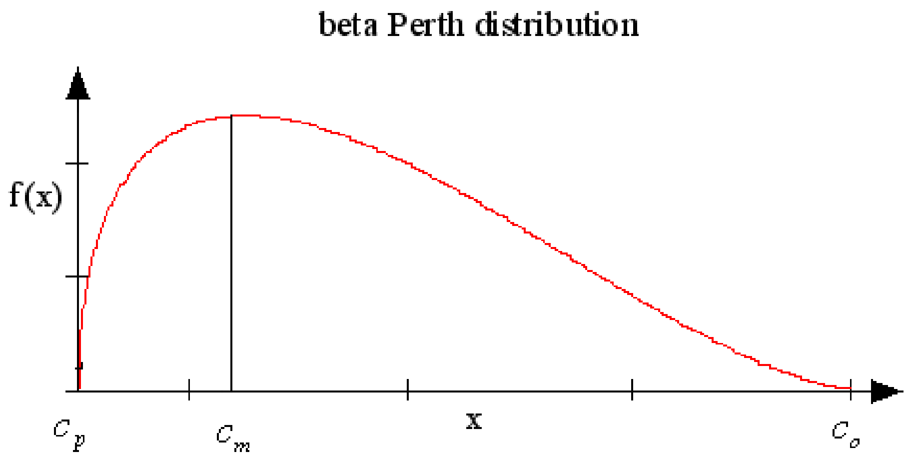
$P(E \mid H)$ - is called the *conditional probability* of seeing the evidence *E* if the hypothesis *H* happens to be true.
It is also called a *likelihood function* when it is considered as a function of *H* for fixed *E*.

$P(E)$ - marginal probability

$P(H \mid E)$ - is called the *posterior probability* of *H* given *E*.

What is the problem?

◆ How to get reliable probabilistic description?



?

Imprecise probability

$$C_i = C_i\left(c_{\min}, c_0, c_{\max}\right)$$

$$R = R\left(c_{\min,0}, c_{mid,0}, c_{\max,0}, \ldots, c_{\min,n}, c_{mid,n}, c_{\max,n}\right)$$

Probability distribution with the interval parameters

Imprecise probability

$$c_{\min,i} \in \left[\underline{c}_{\min,i}, \overline{c}_{\min,i} \right]$$

$$R\left(c_0\right) \in \left[\underline{R}\left(c_0\right), \overline{R}\left(c_0\right) \right]$$

$$R\left(c_0\right) \in \left\{ R(\ldots, c_i, \ldots, c_0) : c_i \in \left[\underline{c}_i, \overline{c}_i \right] \right\}$$

Parallel computing

Monte Carlo silmulation for probabilistic variables.

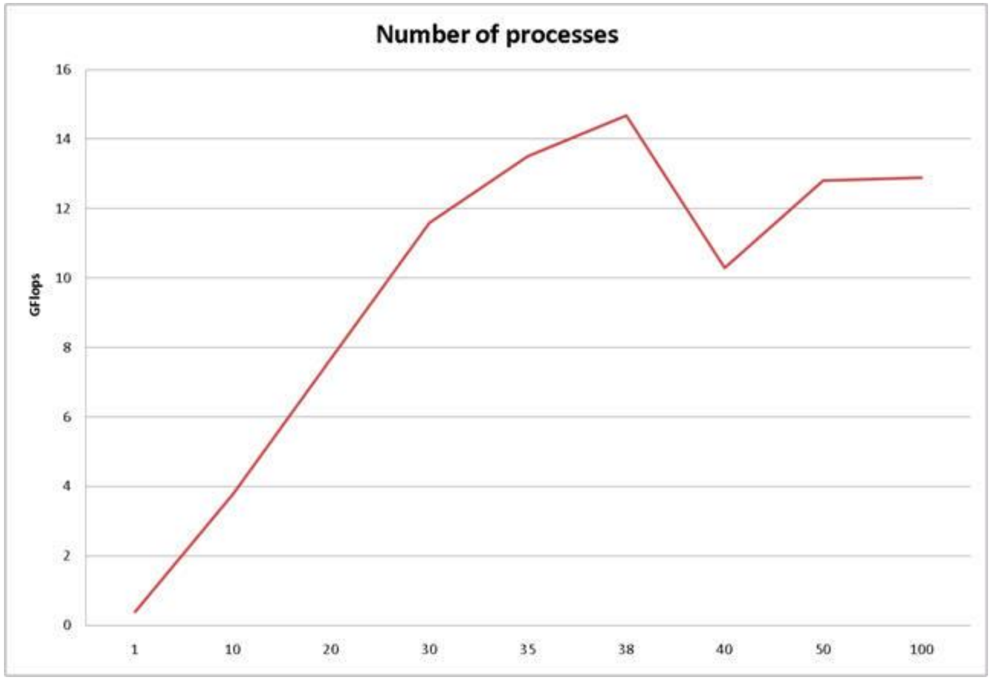
$$R(c_1,\ldots,c_m) \approx \frac{n}{N}$$

Search method for nonprobabilistic parameters

$$R = \left\{ R(c_1,\ldots,c_m) : c_i \in \left\{ c_{i1},\ldots,c_{im} \right\} \right\}$$

MPI_Reduce with MPI_MIN and MPI_MAX

Parallel computing



MPICH2, Windows XP

Windows HPC 2008, MS-MPI

2xDell Precision 690
with 10 cores
24 GB RAM

Parallel algorithm

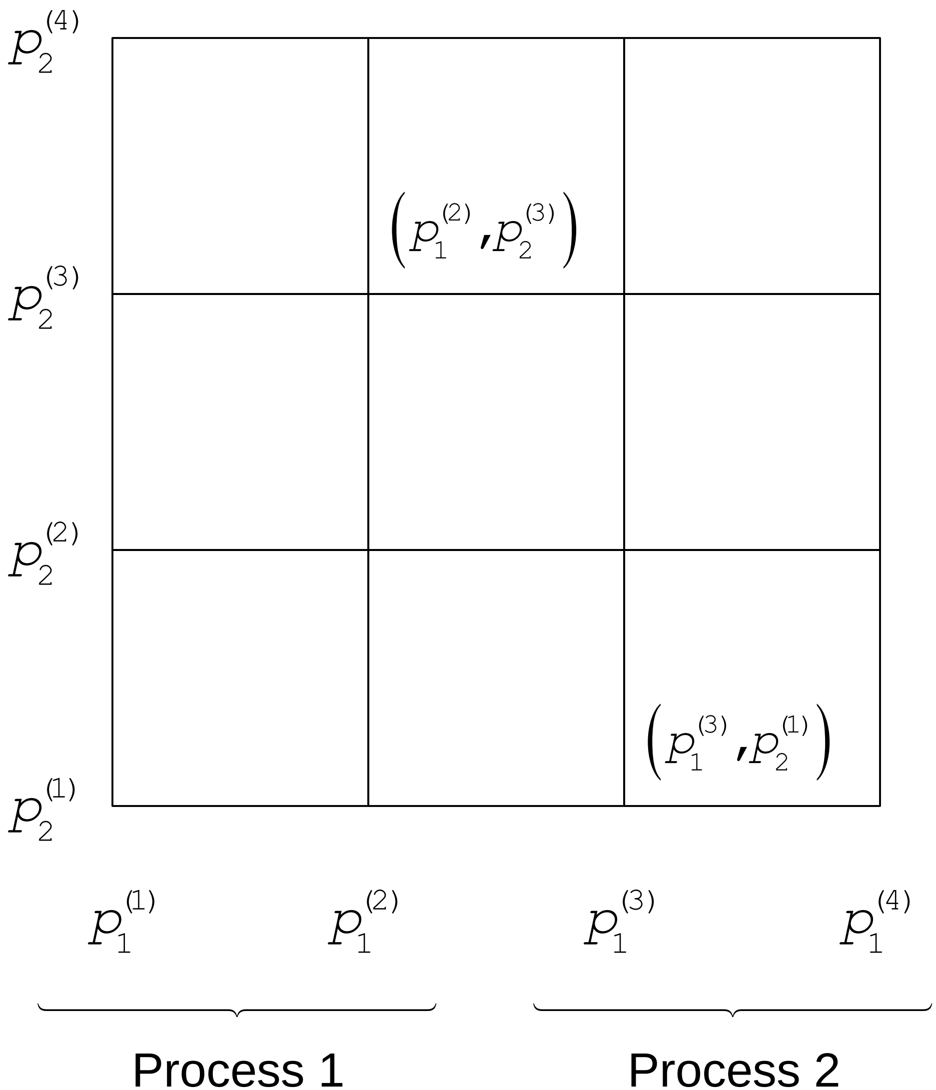
$$p = (p_1, \dots, p_m)$$

Process 1	Process 2	…	Process n
$[\underline{R}_1, \overline{R}_1]$	$[\underline{R}_2, \overline{R}_2]$		$[\underline{R}_n, \overline{R}_n]$
Monte Carlo	Monte Carlo	Monte Carlo	Monte Carlo

$$p_1^{(i)} \in \{p_1^{(1)}, \dots, p_1^{(N)}\} = \bigcup_{j=1}^{n} \{p_1^{(j_1)}, \dots, p_1^{(j_{n_j})}\} = \bigcup_{j=1}^{n} \mathbf{p}_1^{(j)}$$

$\mathbf{p}_1^{(j)}$ - j-*th* process

Search space



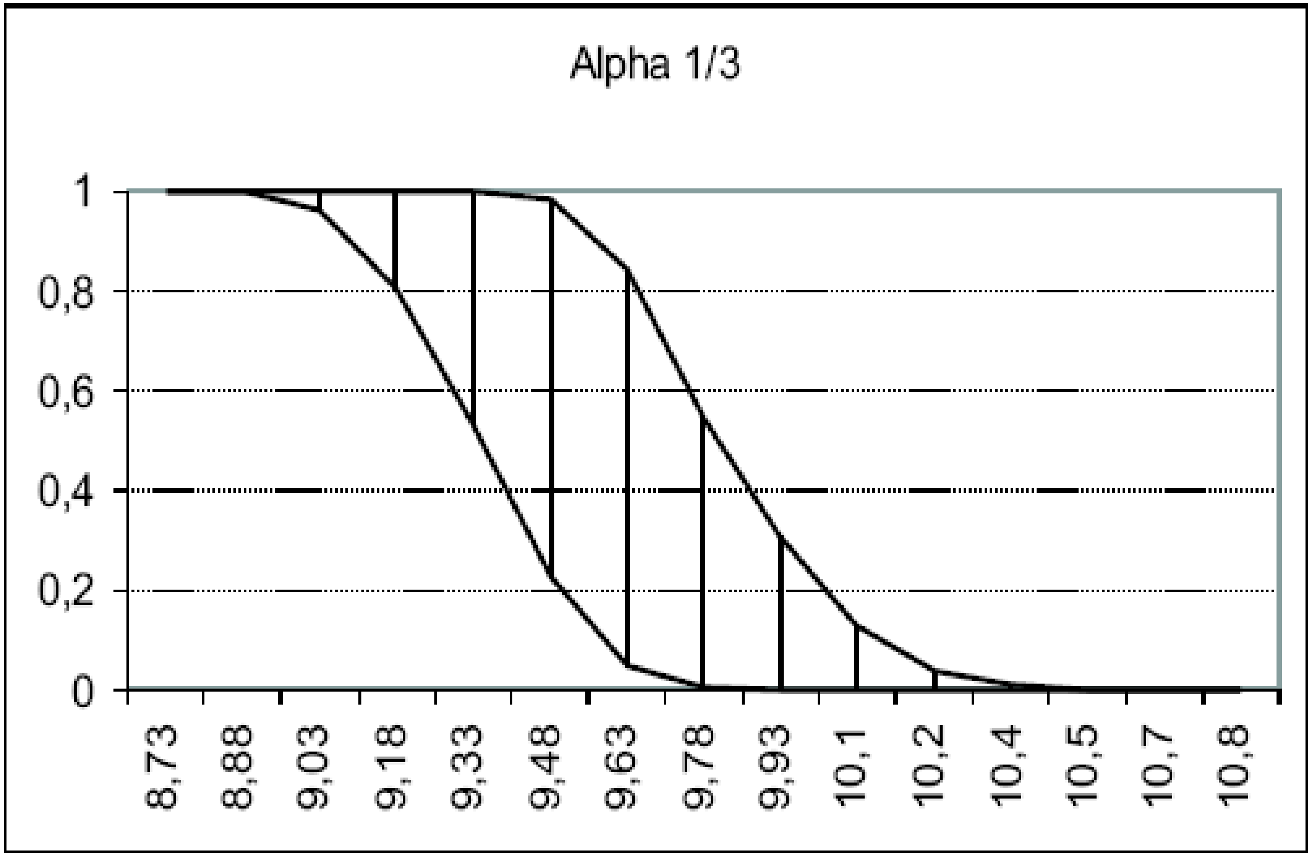
Parallel algorithm

Process 1	Process 2	…	Process n
$[\underline{R}_1, \overline{R}_1]$	$[\underline{R}_2, \overline{R}_2]$		$[\underline{R}_n, \overline{R}_n]$
Monte Carlo	Monte Carlo	Monte Carlo	Monte Carlo

MPI_REDUCE(RMin, RMinAll, M, MPI_DOUBLE, MPI_MMIN, 0, COMM, IERROR)

MPI_REDUCE(RMax, RMaxAll, M, MPI_DOUBLE, MPI_MAX, 0, COMM, IERROR)

Interval risk curve

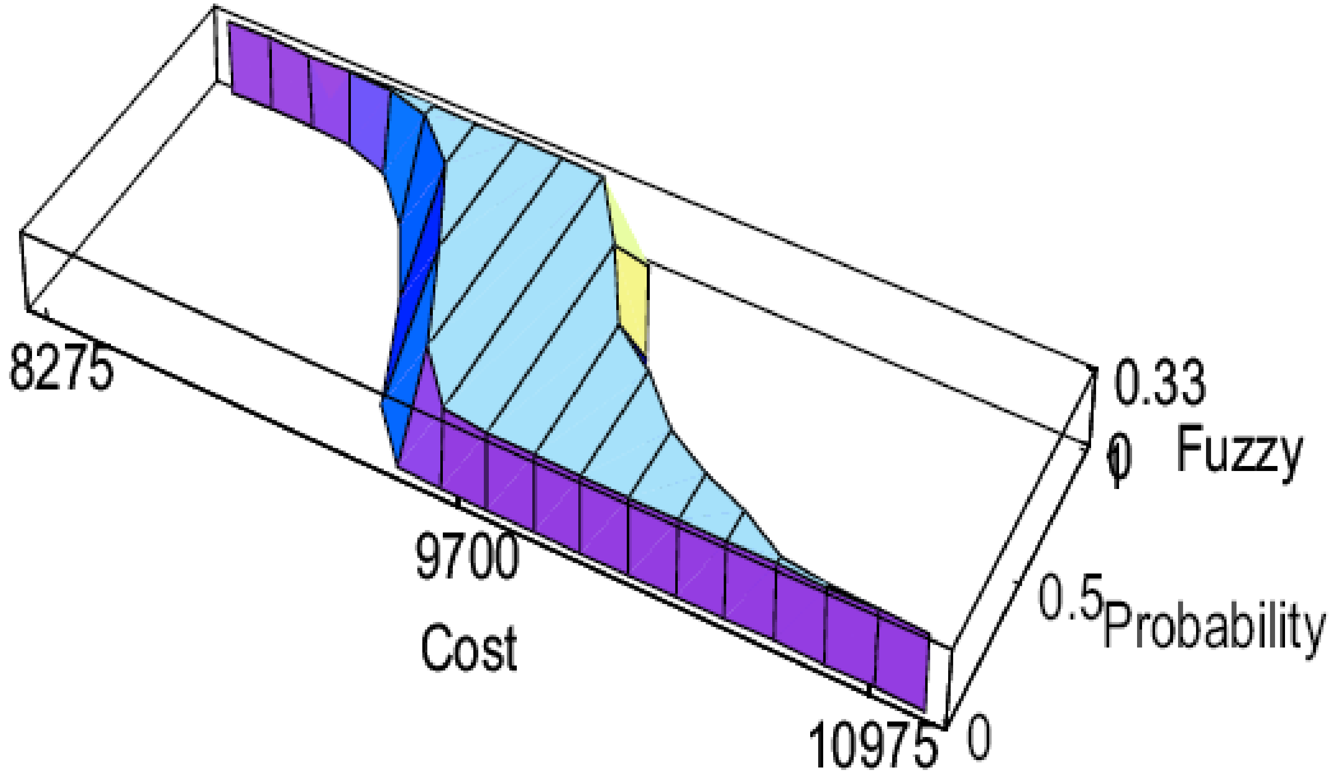


Alpha cut method

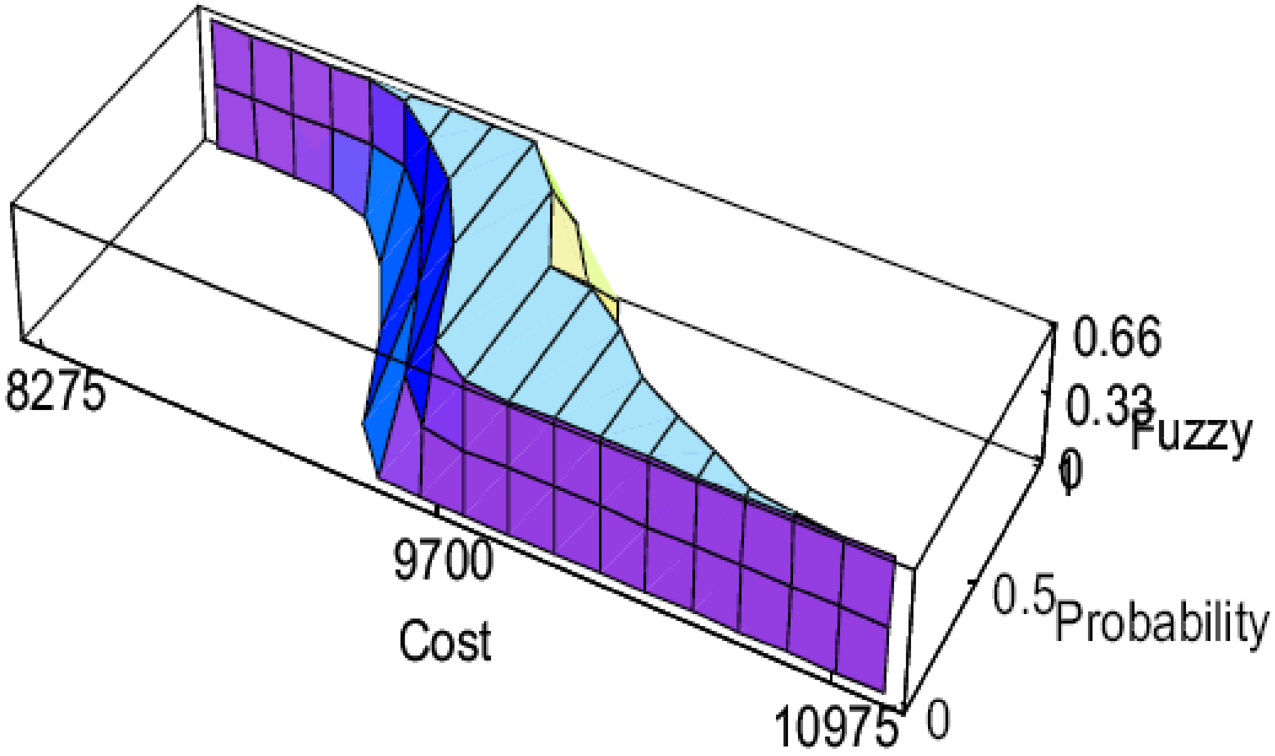
$$\mu_F(x) = \max\left\{\alpha : x \in \left[\underline{F}_\alpha, \bar{F}_\alpha\right]\right\}$$

$$\mu_R(c, p) = \max\left\{\alpha : (p, c) \in R_\alpha\right\}$$

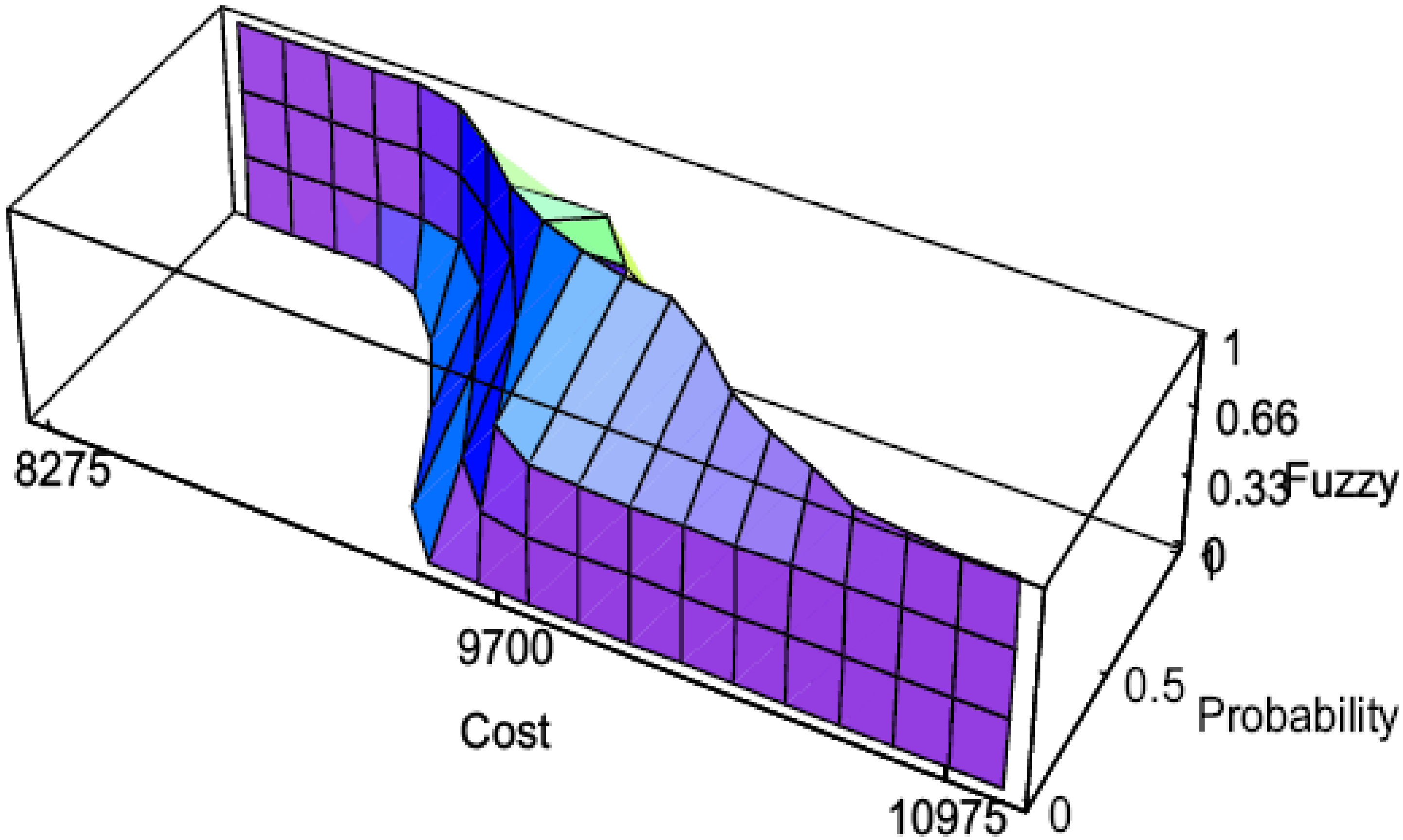
Interval risk curve for different uncertainty



Interval risk curve for different uncertainty



Interval risk curve for different uncertainty



Risk as relation on the ordered set

Financial risk

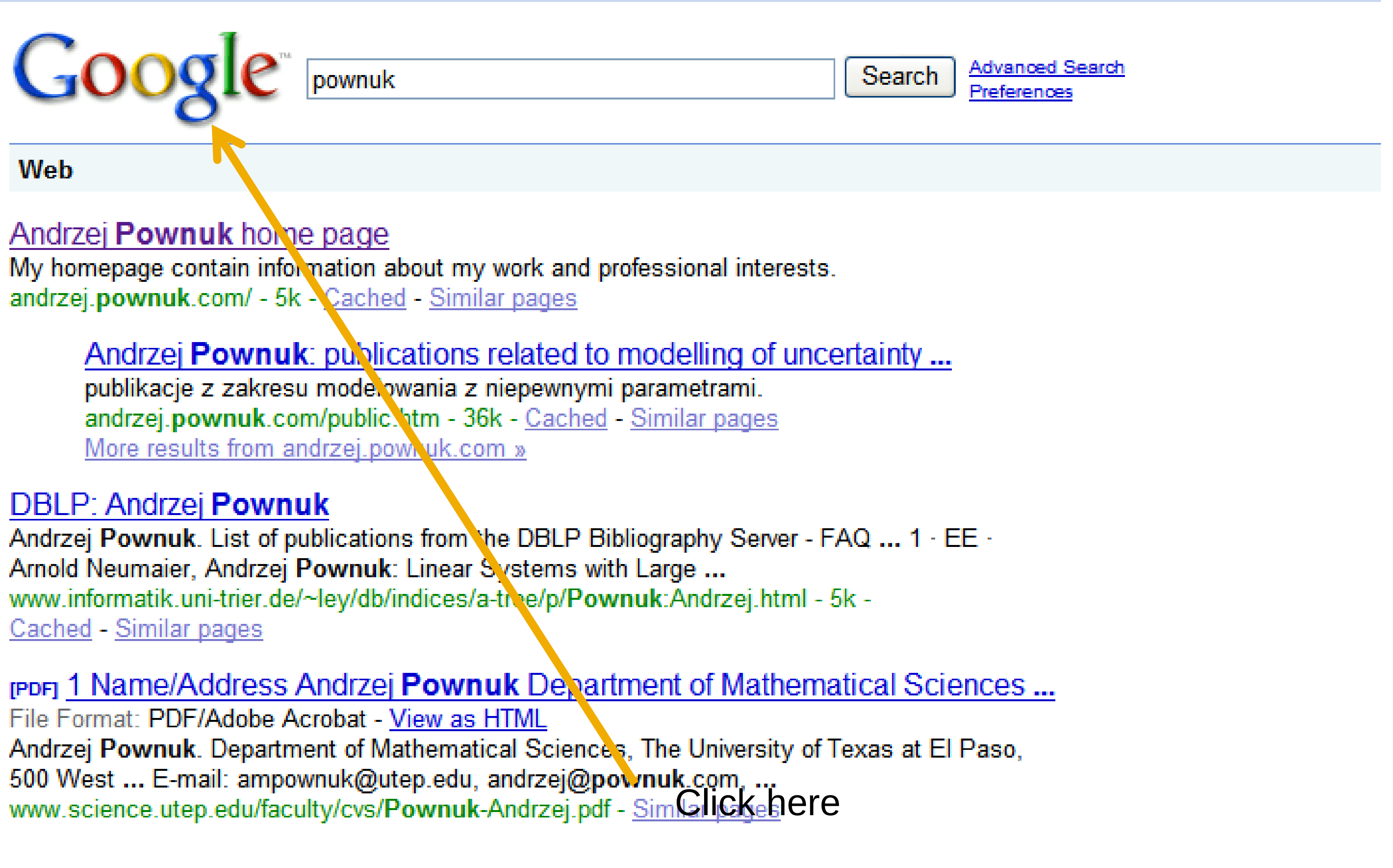
$$P_f = R = P\left\{C > C_0\right\}$$

Probability of failure

$$P_f = R = P\left\{g(x) > 0\right\}$$

Non-probabilistic limit state theory

$$g(x) > 0$$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Web application

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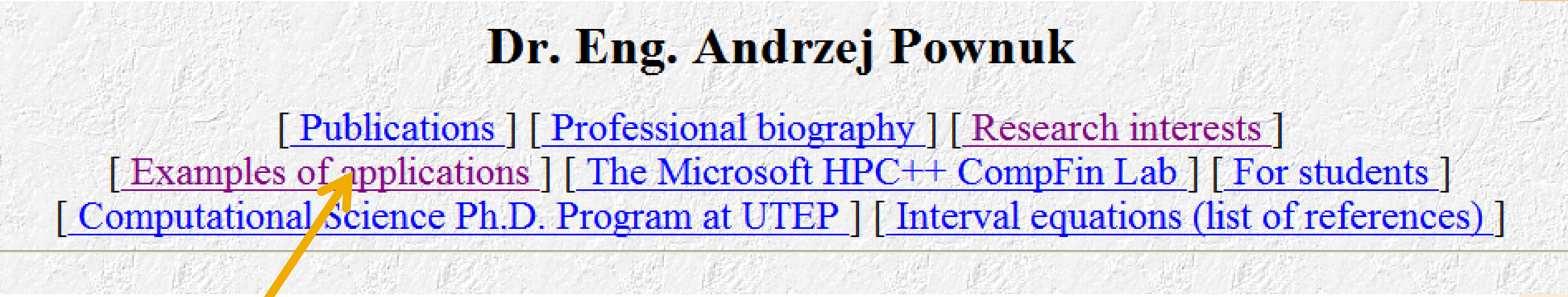
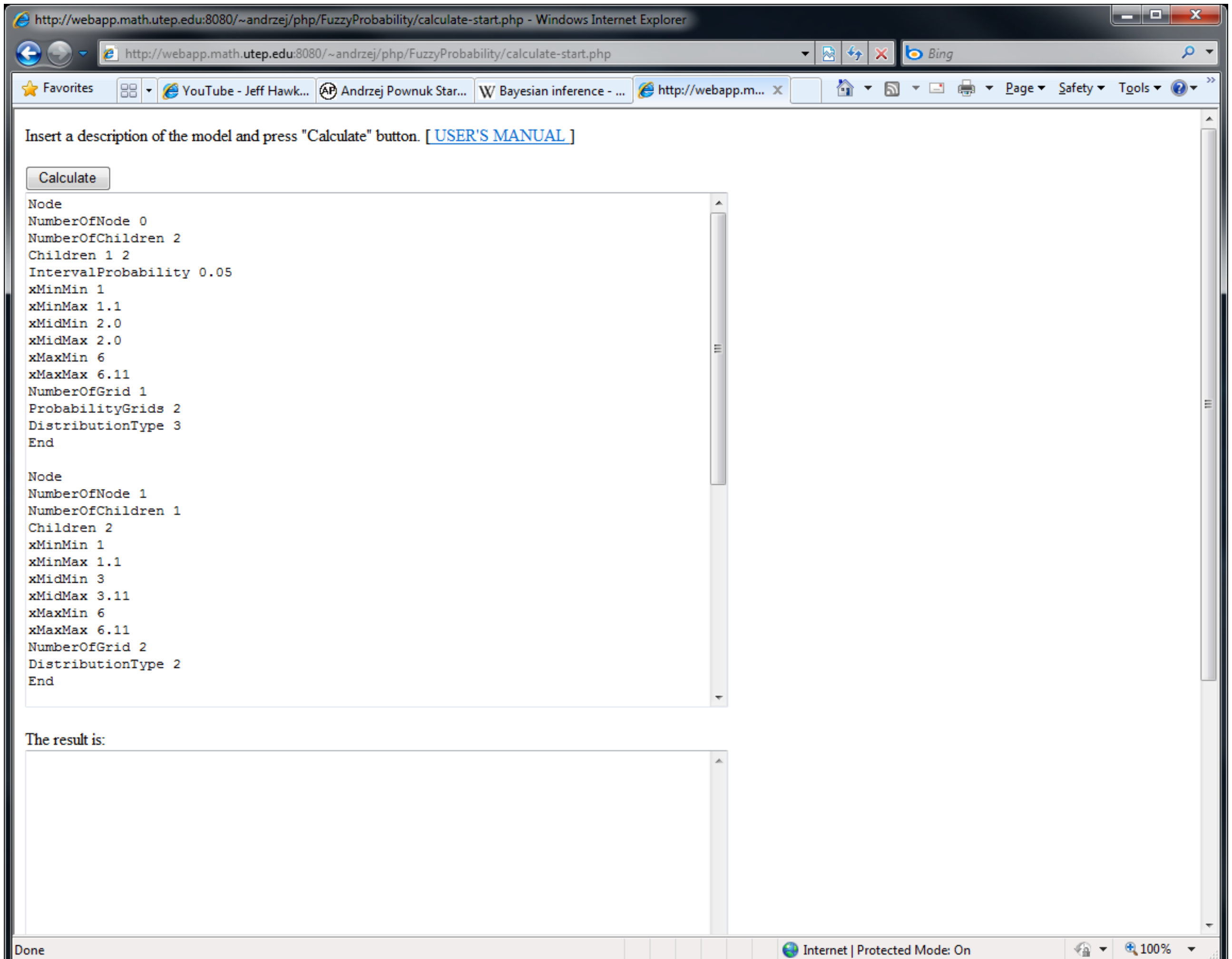


Click here

Web application

http://andrzej.pownuk.com/interval_web_applications.htm



Example script

```
Node
NumberOfNode 0
NumberOfChildren 2
Children 1 2
IntervalProbability 0.05
xMinMin 1
xMinMax 1.1
xMidMin 2.0
xMidMax 2.0
xMaxMin 6
xMaxMax 6.11
NumberOfGrid 1
ProbabilityGrids 2
DistributionType 3
End

Node
NumberOfNode 1
NumberOfChildren 1
Children 2
xMinMin 1
xMinMax 1.1
xMidMin 3
xMidMax 3.11
xMaxMin 6
xMaxMax 6.11
NumberOfGrid 2
DistributionType 2
End

Node
NumberOfNode 2
xMinMin 1
xMinMax 1.1
xMidMin 3
xMidMax 3.11
xMaxMin 6
xMaxMax 6.11
NumberOfGrid 3
DistributionType 1
End

Results
Xmin  0
Xmax  10
NumberOfSimulations 2000
NumberOfClasses  10
NumberOfGrid     2
DistributionType 2
End
```

Example solution

Interval PDF

[0,1] - [0, 0]
[1,2] - [0, 0.0005]
[2,3] - [0, 0.002]
[3,4] - [0.0055, 0.021]
[4,5] - [0.0825, 0.143]
[5,6] - [0.1915, 0.256]
[6,7] - [0.1525, 0.2075]
[7,8] - [0.112, 0.161]
[8,9] - [0.1205, 0.173]
[9,10] - [0.085, 0.136]

Interval CDF

[0,1] - [1, 1]
[1,2] - [0.9995, 1]
[2,3] - [0.998, 1]
[3,4] - [0.978, 0.994]
[4,5] - [0.8385, 0.9065]
[5,6] - [0.6115, 0.7105]
[6,7] - [0.432, 0.533]
[7,8] - [0.3005, 0.3965]
[8,9] - [0.156, 0.2375]
[9,10] - [0.063, 0.116]

Number of iterations : 144 : 144
Number of nodes : 3
Time of calculation : 0.99 [sec]
Time (long) : 1 [sec]
Number of Monte Carlo simulations : 2000

0 distribution type : 3 grid : 1
1 distribution type : 2 grid : 2
2 distribution type : 1 grid : 3

Parameter dependent probabilistic characteristics

$$R = P\left\{C > C_0\right\} = R(p)$$

$$\underline{R} = \min\{R(p) : p \in \mathbf{p}\}$$

$$\overline{R} = \min\{R(p) : p \in \mathbf{p}\}$$

Optimization

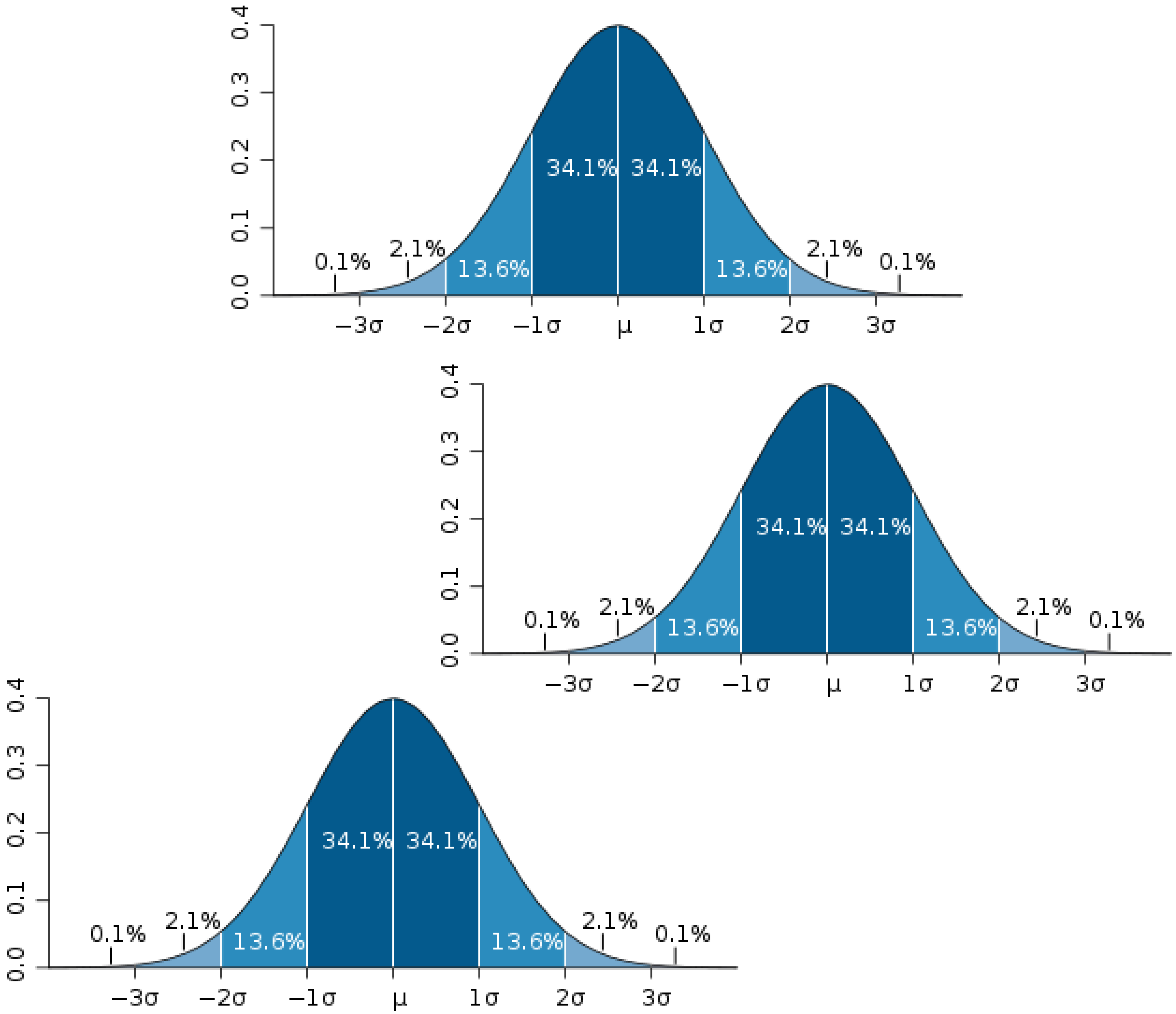
- Powell method (gradient free optimization)

$$\frac{\partial R}{\partial p_i} \approx \frac{u\left(p_i + \Delta p_i\right) - u\left(p_i - \Delta p_i\right)}{2\Delta p_i}$$

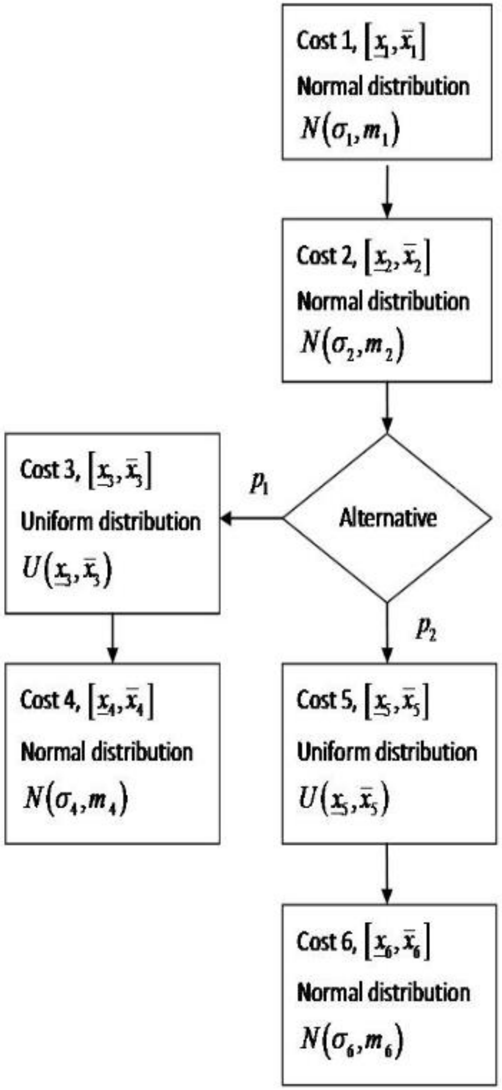
$$\underline{R} = \min\{R(p) : p \in \mathbf{p}\}$$

$$\overline{R} = \min\{R(p) : p \in \mathbf{p}\}$$

Conflicting data



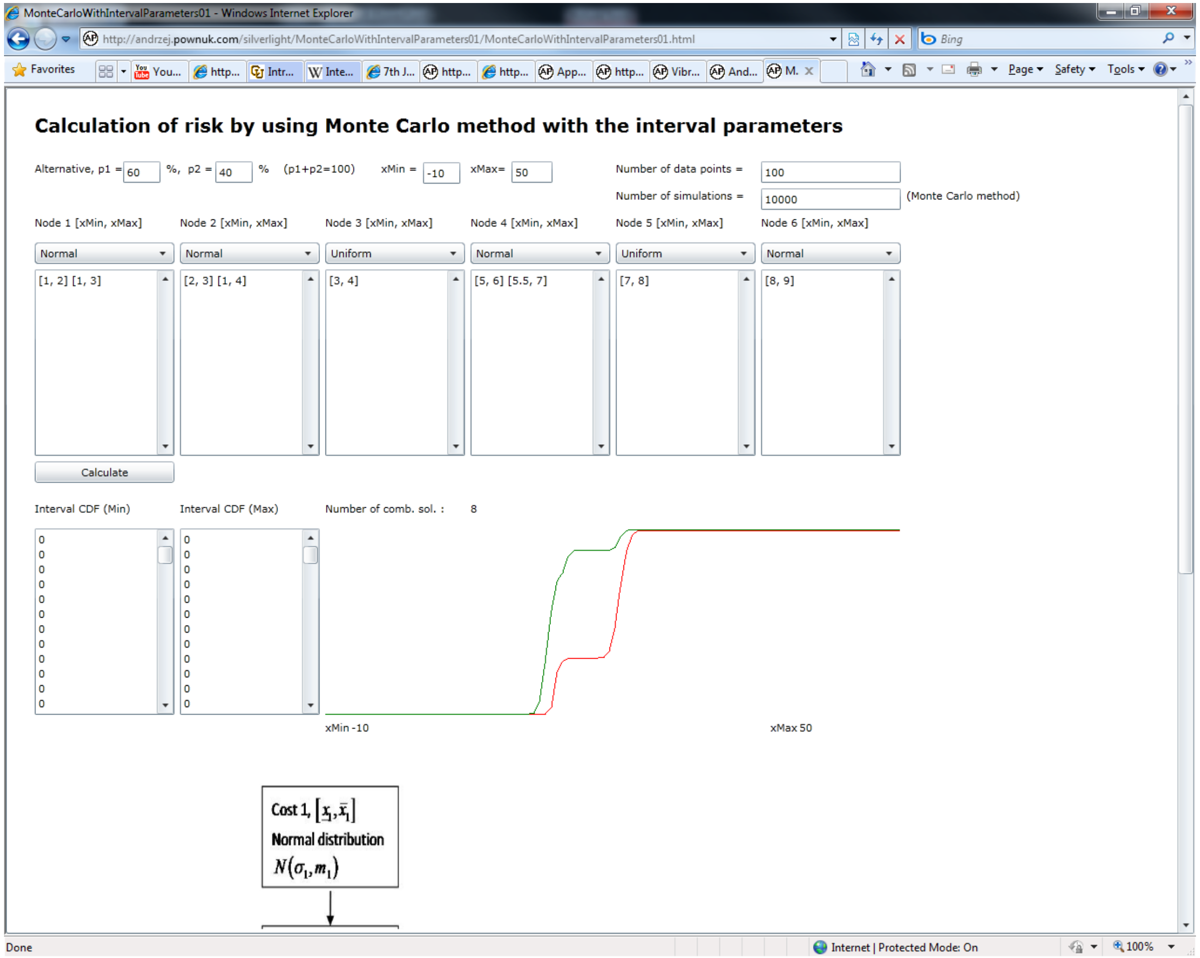
Example 2



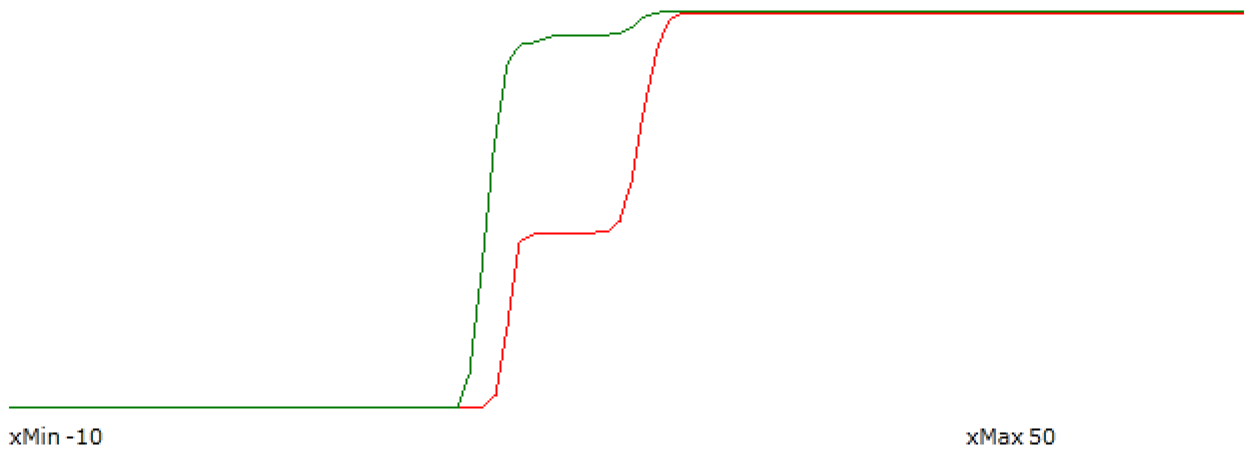
Cost 1, $[\underline{x}_1, \bar{x}_1]$
Normal distribution
$N(\sigma_1, m_1)$
Cost 2, $[\underline{x}_2, \bar{x}_2]$
Normal distribution
$N(\sigma_2, m_2)$
Alternative
p_1
p_2
Cost 3, $[\underline{x}_3, \bar{x}_3]$
Uniform distribution
$U(\underline{x}_3, \bar{x}_3)$
Cost 4, $[\underline{x}_4, \bar{x}_4]$
Normal distribution
$N(\sigma_4, m_4)$
Cost 5, $[\underline{x}_5, \bar{x}_5]$
Uniform distribution
$U(\underline{x}_5, \bar{x}_5)$
Cost 6, $[\underline{x}_6, \bar{x}_6]$
Normal distribution
$N(\sigma_6, m_6)$

Calculation of the interval risk

http://andrzej.pownuk.com/silverlight/MonteCarloWithIntervalParameters01/MonteCarloWithIntervalParameters01.html



Numerical results



Parallel algorithm

Process 1	Process 2	…	Process n
$[\underline{R}_1, \overline{R}_1]$	$[\underline{R}_2, \overline{R}_2]$		$[\underline{R}_n, \overline{R}_n]$
Monte Carlo	Monte Carlo	Monte Carlo	Monte Carlo

$$\text{number of simulations} = m_1 \cdot m_2 \cdot \ldots \cdot m_n$$

If $i_{\min_j} \le i \le i_{max_j}$ execute in process "*j*"

Parallel algorithm

Process 1	Process 2	…	Process n
$[\underline{R}_1, \overline{R}_1]$	$[\underline{R}_2, \overline{R}_2]$		$[\underline{R}_n, \overline{R}_n]$
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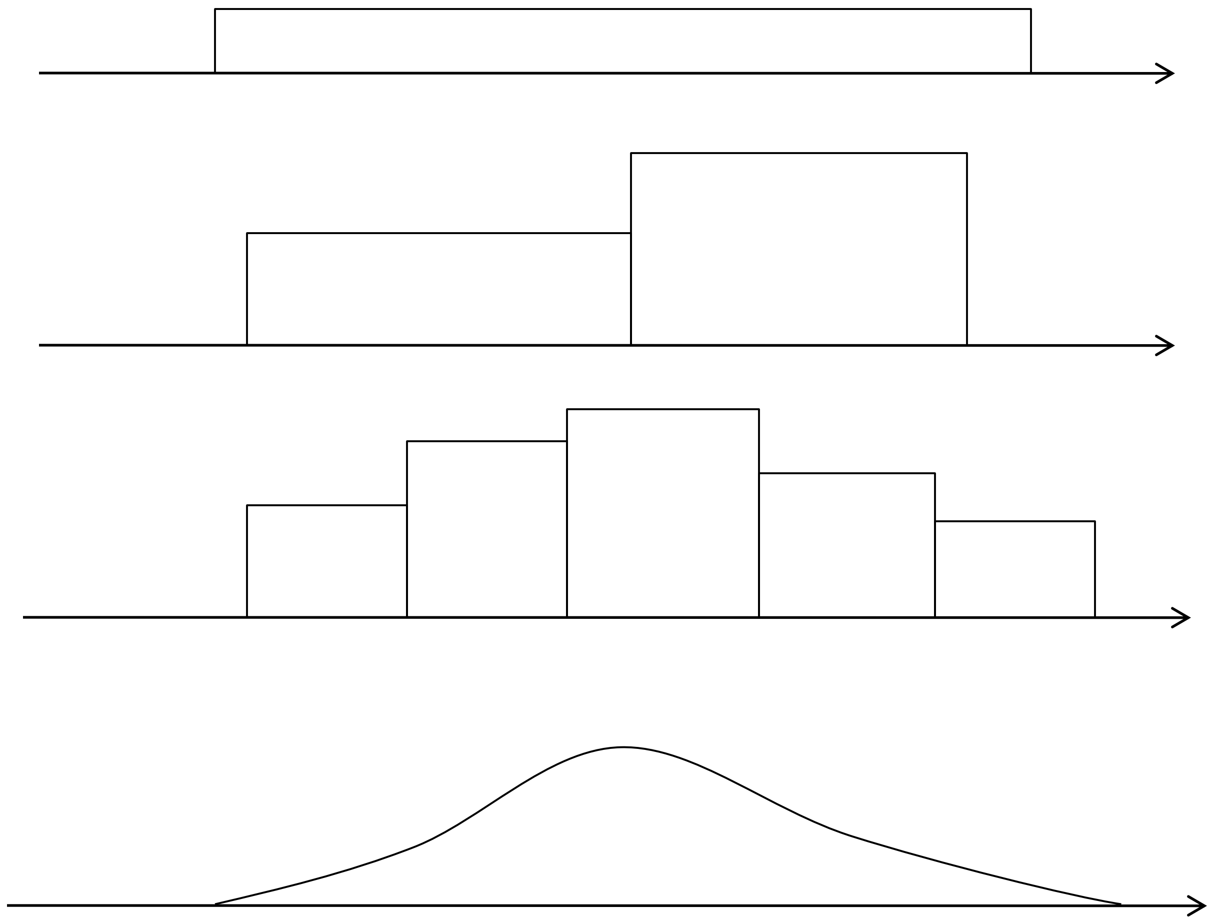
MPI_REDUCE(RMax, RMaxAll, M, MPI_DOUBLE, MPI_MAX, 0, COMM, IERROR)

More information



Number of bins and width

$$k = \left\lceil \frac{\max x - \min x}{h} \right\rceil$$

$$k = \lceil \log_2 n + 1 \rceil$$ Sturges' formula

which implicitly bases the bin sizes on
the range of the data, and can perform poorly if $n<30$

Conclusions

- It is possible to use imprecise probability in order to model the risk of engineering project
- In the first approach PDF with the interval parameters was applied
- In the second approach several conflicting data was modeled by using different PDF
- In both cases the results can be described by using the concept of upper and lower probability
- HPC computing significantly speed up the calculations